MIDTERM, MATH 430, DUE THU MAR 15TH

All solutions should be with proofs, you may quote from the book or from previous home works

(1) If G is a finite abelian group with $n|o(G)$, show that number of solutions of $x^n = e$ in G is a multiple of n.

(2) As usual, for a subgroup H of G, we write $N(H)$ to be the normalizer of H in G, $H = \{g \in G | gHg^{-1} \subset H\}$. If P is a p-Sylow subgroup of a finite group G, show that $N(N(P)) = N(P)$.

(3) If for an $a \in G$, G any group, one can solve the equation $x^2ax = a^{-1}$, show that $a = b^3$ for some $b \in G$.

(4) If G is a group of order 385, show that its 11-Sylow subgroup is normal and its 7-Sylow subgroup is in the center.

(5) Let $G = \mathbb{Z}/2\mathbb{Z} = \{e, \sigma\}$, act on $\mathbb{F}_p v_1 + \mathbb{F}_p v_2$, a vector space of dimension 2 with basis v_1, v_2 where p is a prime, by $\sigma(v_1) = v_2, \sigma(v_2) = v_1$. Calculate the number of distinct orbits.

(6) Calculate the number of distinct group homomorphisms from $\mathbb{Z}/4\mathbb{Z}$ to $\mathbb{Z}/10\mathbb{Z} \times \mathbb{Z}/16\mathbb{Z}$.