Qualifying Exam, Math 432, May 7th, 2005
Time 1hr

Answer any four questions. You may quote results from class as needed unless a result from class is what you are expected to prove. Please write legibly.

1. Let G be a finite group and n a positive integer relatively prime to the order of G. Then prove that the map $G \to G$ given by $x \mapsto x^n$ is bijective.

2. Prove that if R is a Unique factorisation domain, then so is $R[X]$.

3. Given two odd integers a, b, show that you can find an integer n such that $n \equiv a \bmod 34$ and $n \equiv b \bmod 54$.

4. Show that the polynomial $X^p - X + 1$ is irreducible over $\mathbb{F}_p$.

5. Let $S \subset \mathbb{Z}$ be a multiplicatively closed subset and let $M = \mathbb{Q}/S^{-1}\mathbb{Z}$. Find necessary and sufficient conditions on S so that $\operatorname{Ass} M$ is finite.

6. Give an example with proof of two non-zero modules M, N over a commutative ring with 1 so that $M \otimes N = 0$.

7. Compute the character for the standard representation of S_4 over $\mathbb{C}$ and prove that it is faithful.