Homework 2, Math 5032, Due Feb 4th

1. Compute the Galois group of the polynomials $X^3 - X - 1$ and $X^4 - a$ where $a \in \mathbb{Z}$ not square free, over $\mathbb{Q}$.

2. Compute the Galois group of the polynomial $X^3 + X + t$ over $\mathbb{C}(t)$, the rational functions in t.

3. Let $f(X) \in \mathbb{Q}[X]$ be a monic polynomial of degree n and let K be its splitting field. Assume that the Galois group is S_n. Show that f is irreducible. If α is a root of f show that there are no nontrivial automorphism of $\mathbb{Q}(\alpha)$.

4. Let p be any prime. Show that the polynomial $f(X) = X^5 - p^2 X + p \in \mathbb{Q}[X]$ is irreducible. Show that the Galois group of $f(X)$ over $\mathbb{Q}$ is S_5.

5. Let E/K be a (finite) Galois extension with Galois group G and let L be an intermediate field. Let $H \subset G$ be the Galois group of E over L. Let $N \subset G$ be the set of all $\sigma \in G$ such that $\sigma(L) = L$. Show that N is the normalizer of H in G.