

Algebraic Geometry, Math 539, Homework 3

1. Let $f : X \rightarrow Y$ be a proper morphism with $f_*\mathcal{O}_X = \mathcal{O}_Y$. Then show that the natural map $f^* : \text{Pic } Y \rightarrow \text{Pic } X$ is injective. Construct an example to show that the condition on the direct image of the structure sheaf is necessary.
2. If X, Y are projective varieties, show that $N^1(X) \times N^1(Y)$ is naturally a subgroup of $N^1(X \times Y)$. Show that if $X = Y$ is a smooth curve, then these two groups are equal if and only if it is \mathbb{P}^1 .
3. Let $f : X \rightarrow Y$ be the blowing up of a smooth point of Y . Then show that $N^1(X)$ is naturally equal to $N^1(Y) \oplus \mathbb{Z} \cdot E$ where E is the exceptional divisor.
4. Verify details of example 1.35 in the book.