Algebraic Geometry, Math 539, Homework 4

1. A subset $Y \subset X$ of a scheme is called *constructible* if it is a finite union of locally closed subsets. Show that if $f : X \to Y$ is a morphism of quasi-projective varieites, then $f(X)$ is constructible. (This is known as Chevalley's Theorem).

2. Show that an open immersion is flat. (A morphism $f : X \to Y$ is flat if $\mathcal{O}_X$ is flat over Y).

3. Let $f : X \to Y$ be as above and assume that f is flat. Show that f is an open map.

4. Let $X = \mathbb{A}^1$ and $Y = \mathbb{A}^2$ and let $f : X \to Y$ be given by $t \mapsto (t^n, t^{n+1})$, for some n. Describe the flattening stratification of Y for the sheaf $\mathcal{O}_X$, for $n = 2, 3$.

5. Find the smallest integer m so that $T_{\mathbb{P}^n}$ is m-regular, where $T_{\mathbb{P}^n}$ is the tangent bundle of $\mathbb{P}^n$.

6. Let $X \subset \mathbb{P}^n$ be a smooth projective curve of degree d and genus g. Given an integer r find the smallest integer m so that for any line bundle L on X of degree r, L is m-regular.