Math 131, Spring 2004
Quiz #2, Discussion Section B (Tuesday, 12:00-1:00)

Quiz problems should be solved using the methods discussed in this course. A calculator is not permitted. To receive full credit, show enough work to make it clear how you got your answer.

Name: Answer Key ID# 2/3/04

1. Use the Squeeze Theorem to show that $\lim_{x\to 0} x^2 \sin\left(\frac{2\pi}{x}\right) = 0$. Please be sure to verify the hypotheses of the theorem.

Squeeze Thm: ① If $f(x) \le g(x) \le h(x)$ for x near a (except possibly at a) & ② $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$, then $\lim_{x\to a} g(x) = L$.

We know $-1 \le \sin\left(\frac{2\pi}{x}\right) \le 1$.

So $-x^2 \le x^2 \sin\left(\frac{2\pi}{x}\right) \le x^2$. (hyp. ①)

Now $\lim_{x\to 0} (-x^2) = 0$ & $\lim_{x\to 0} x^2 = 0$. (hyp. ②)

Thus, $\lim_{x\to 0} x^2 \sin\left(\frac{2\pi}{x}\right) = 0$.

2. Evaluate $\lim_{x\to 1} \frac{x-1}{|x-1|}$ if it exists. If it does not exist, then please explain why.

$$\frac{x-1}{|x-1|} = \begin{cases} \frac{x-1}{-(x-1)} & x \le 1 \\ \frac{x-1}{(x-1)} & x \ge 1 \end{cases}$$

So $\lim_{x\to 1^+} \frac{x-1}{|x-1|} = \lim_{x\to 1^+} \frac{x-1}{x-1} = 1$ &

$\lim_{x\to 1^-} \frac{x-1}{|x-1|} = \lim_{x\to 1^-} \frac{x-1}{(-x+1)} = -1$

So $\lim_{x\to 1^+} \frac{x-1}{|x-1|} \ne \lim_{x\to 1^-} \frac{x-1}{|x-1|} \Rightarrow \lim_{x\to 1} \frac{x-1}{|x-1|}$ DNE.

Math 131, Spring 2004
Quiz #2, Discussion Section C (Thursday, 12:00-1:00)
AND A

Quiz problems should be solved using the methods discussed in this course. A calculator is not permitted. To receive full credit, show enough work to make it clear how you got your answer.

Name: Answer Key ID#: 2/5

1. Find the value(s) of A that will make $g(x)$ continuous on $(-\infty, \infty)$, where

$$g(x) = \begin{cases} A^2x - 7A & \text{if } x \geq 2 \\ 2x & \text{if } x < 2. \end{cases}$$

$g(x)$ is clearly continuous on $(-\infty, 2)$ & $(2, \infty)$; we need to ensure continuity at 2.

So we want $g(2) = \lim_{x\to 2^+} g(x) = \lim_{x \to 2^-} g(x)$.

$\lim_{x\to 2^+} g(x) = 2A^2 - 7A$, $\lim_{x\to 2^-} g(x) = 4$, & $g(2) = 2A^2 - 7A$

So we want $2A^2 - 7A = 4 \iff 2A^2 - 7A = 4 = 0$
$\iff (2A+1)(A-4) = 0$
$\iff A = -1/2$ or $A = 4$

2. If $\lim_{x\to 1} f(x) = -2$ and $\lim_{x \to 1} g(x) = 5$, then please evaluate

$$\lim_{x\to 1}\left[\frac{(x^2+6x-7)\, f(x)}{(2x^2-5x+3)\, g(x)}\right].$$

$$\lim_{x\to 1}\left[\frac{(x^2+6x-7)f(x)}{(2x^2-5x+3)g(x)}\right] = \lim_{x\to 1}\frac{(x^2+6x-7)}{(2x^2-5x+3)} \cdot \lim_{x\to 1}\frac{f(x)}{g(x)}$$

$$= \lim_{x\to 1}\frac{(x+7)(x-1)}{(2x-3)(x-1)} \cdot \frac{\lim_{x\to 1} f(x)}{\lim_{x\to 1} g(x)}$$

$$= \lim_{x\to 1}\frac{(x+7)}{(2x-3)} \cdot \frac{-2}{5}$$

$$= \frac{8}{-1} \cdot \frac{-2}{5}$$

$$= \frac{16}{5}$$