Math 131, Spring 2004
Quiz #4, Discussion Section A (Thursday, 11:00-12:00)

Quiz problems should be solved using the methods discussed in this course. A calculator is not permitted. To receive full credit, show enough work to make it clear how you got your answer.

Name: Answer Key ID#

1. Where is the tangent line to $f(x) = 2x^3 + 3x^2 - 12x + e^{\pi}$ horizontal?

$f(x) = 2x^3 + 3x^2 - 12x + e^{\pi}$

$f'(x) = 6x^2 + 6x - 12$

So $f'(x) = 0 \iff 6x^2 + 6x - 12 = 0 \iff x^2 + x - 2 = 0$
$\iff (x+2)(x-1) = 0 \iff x = -2$ or $x = 1$

So the tangent line is horizontal at $x = -2$ & $x = 1$.

2. Sketch a graph of a function with the following properties:

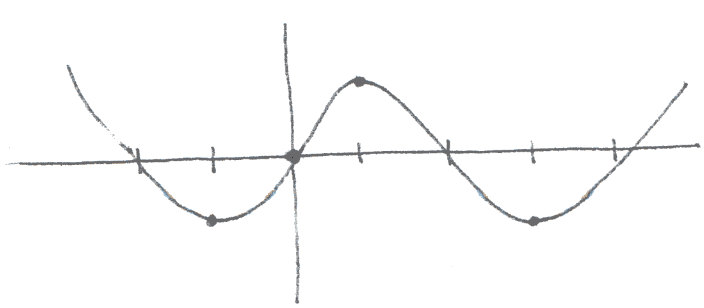
(i). $f(0) = 0$ and $f'(-1) = f'(1) = f'(3) = 0$
(ii). $f'' > 0$ on $(-\infty, 0)$ and $(2, \infty)$ and $f'' < 0$ on $(0, 2)$.
(iii). $f' > 0$ on $(-1, 1)$ and $(3, \infty)$ and $f' < 0$ on $(-\infty, -1)$ and $(1, 3)$

(i) $\Rightarrow$ tan. line horizontal at $x = -1$, $x = 1$, & $x = 3$

(ii) $\Rightarrow$ $\left.\begin{array}{l} f \text{ c.c.} \uparrow \text{ on } (-\infty, 0) \text{ \& } (2, \infty) \\ f \text{ c.c.} \downarrow \text{ on } (0, 2) \end{array}\right\} \Rightarrow$ f has an inflection pnt at $x = 0$ & $x = 2$

(iii) $\Rightarrow$ f increasing on $(-1, 1)$, $(3, \infty)$
f decreasing on $(-\infty, -1)$, $(1, 3)$

So f has a min at $x = -1$, a max at $x = 1$, & a min at $x = 3$



Math 131, Spring 2004
Quiz #4, Discussion Section B (Tuesday, 12:00-1:00)

Quiz problems should be solved using the methods discussed in this course. A calculator is not permitted. To receive full credit, show enough work to make it clear how you got your answer.

Name: Answer Key ID#

1. A particle is moving along the x-axis. At time t seconds, its velocity is $v(t) = \sqrt{2t+5}$. When $t = 2s$, its position is 9 meters. Use linear approximation to estimate the position of the particle $\frac{1}{4}$s later. Is this estimate an overestimate or an underestimate? Why?

Let $p(t)$ denote the position function. We are given $p(2)=9$. Since $p'(t)=v(t)$, $p'(2)=v(2)=\sqrt{9}=3$. So the tan. line to $p(t)$ at 2 is $y-9=3(x-2)$ or $y=3x+3$. So our linear approx. function is $L(x)=3x+3$. We estimate $p(2.25)\approx L(2.25)=3(2.25)+3=9.75$.

Now $p'(t)=v(t)$ is increasing at $t=2$, so $p''(t)>0$ at $t=2$. Therefore, p is concave up at $t=2$. So the tangent line lies under the curve & our estimate is an underestimate.

2. Draw a graph of a function such that the first derivative changes sign exactly once and the second derivative is always negative. To help you with this, please answer the following questions.

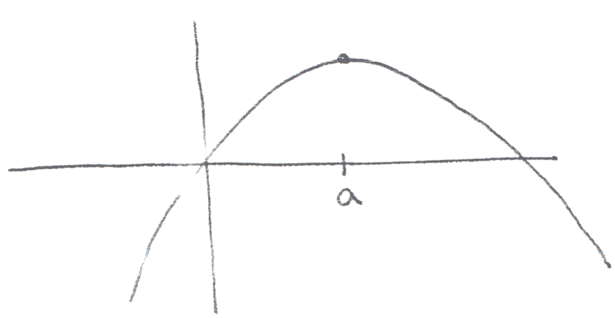
a) If f' changes sign at $x = a$, then what can we say about $f(a)$?

f has a horizontal tangent at $x=a$.

b) If f'' is always negative, then what can we say about f?

f is always concave down

c) Sketch an example of a function with the above two properties.



Math 131, Spring 2004
Quiz #4, Discussion Section C (Thursday, 12:00-1:00)

Quiz problems should be solved using the methods discussed in this course. A calculator is not permitted. To receive full credit, show enough work to make it clear how you got your answer.

Name: Answer Key ID# ______________

1. Please answer the following true/false questions, and explain your reasoning. (Your explanation can consist of a picture.)

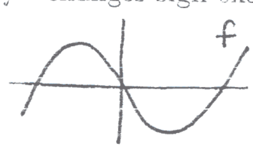
a) True/False: It is possible for a function f to be concave down and have its derivative f' be an increasing function. f c.c. $\downarrow \Rightarrow f'' < 0$

f' inc. $\Rightarrow f'' > 0$

So false

b) True/False: If f'' changes sign exactly once, then f must be a decreasing function.

False :



For this f, f is c.c. $\downarrow$ on $(-\infty, 0)$ & c.c. $\uparrow$ on $(0, \infty)$, so f'' changes sign exactly once at $x=0$, but f is not decreasing

c) True/False: If f' changes sign at $x = a$ and f'' is always positive, then $x = a$ is a local minimum.

$f'' > 0 \Rightarrow f$ c.c. $\uparrow$

$\Rightarrow a$ is a local min.

So True.

2. Compute the following:

a) (1 point) $\frac{d}{dx}(x^e) = ex^{e-1}$

b) (2 points) $\frac{d}{dx}\left(\frac{4x^2 - 3x + 3\sqrt{x}}{x}\right) = \frac{d}{dx}(4x - 3 + 3x^{-1/2})$

$= 4 - \frac{3}{2}x^{-3/2}$