Math 131, Spring 2004
Quiz #6, Discussion Section A (Thursday, 11:00-12:00)

Quiz problems should be solved using the methods discussed in this course. A calculator is not permitted. To receive full credit, show enough work to make it clear how you got your answer.

Name: KEY ID#

1. Differentiate $f(x) = 6^{-1/t}(\tan x)$. Please simplify your work as much as you can.

Written as such, x is the variable, so t is a constant.

$$f'(x) = \underbrace{6^{-1/t}}_{=\text{constant}}(\tan x)' = 6^{-1/t}\sec^2 x$$

A GOOD PROBLEM : $f(x) = 6^{-1/x}(\tan x)$

use Chain Rule!

$$f'(x) \overset{PR}{=} 6^{-1/x}(\tan x)' + \tan x\,(6^{-1/x})'$$
$$= 6^{-1/x}\sec^2 x + \tan x\,(6^{-1/x})(-\tfrac{1}{x})'$$
$$= 6^{-1/x}\sec^2 x + \tan x\,(6^{-1/x})(\tfrac{1}{x^2})$$
$$= 6^{-1/x}[\sec^2 x + \tan x/x^2]$$

2. Use implicit differentiation to find $\frac{dy}{dx}$ of $x^4 + y^4 = 1 + ye^x$.

$$\frac{d}{dx}(x^4 + y^4) = \frac{d}{dx}(1 + ye^x)$$

$$4x^3 + \underbrace{4y^3\frac{dy}{dx}}_{\text{we used the chain rule}} = 0 + \underbrace{ye^x + e^x\frac{dy}{dx}}_{\text{we used the product rule}}$$

$$4y^3\frac{dy}{dx} - e^x\frac{dy}{dx} = ye^x - 4x^3$$

$$\frac{dy}{dx}(4y^3 - e^x) = ye^x - 4x^3$$

$$\frac{dy}{dx} = \frac{ye^x - 4x^3}{4y^3 - e^x}$$

Math 131, Spring 2004
Quiz #6, Discussion Section B (Tuesday, 12:00-1:00)

Quiz problems should be solved using the methods discussed in this course. A calculator is not permitted. To receive full credit, show enough work to make it clear how you got your answer.

Name: KEY ID#: ____________

1. Find the derivative of $f(x) = x^2 \cos\left(\sqrt{2-5x}\right)$. Please simplify as much as you can.

$$f'(x) \overset{PR}{=} x^2 \underbrace{\left[\cos\left(\sqrt{2-5x}\right)\right]'}_{\text{use chain rule}} + \cos\left(\sqrt{2-5x}\right)(x^2)'$$

$$= x^2\left(-\sin\left(\sqrt{2-5x}\right)\right)\underbrace{\left(\sqrt{2-5x}\right)'}_{\text{chain rule}} + \cos\left(\sqrt{2-5x}\right)(2x)$$

$$= x^2\left(-\sin\left(\sqrt{2-5x}\right)\right)\left(\tfrac{1}{2}(2-5x)^{-\frac{1}{2}}\right)(-5) + 2x\cos\left(\sqrt{2-5x}\right)$$

$$= \frac{5x^2\sin\left(\sqrt{2-5x}\right)}{2\sqrt{2-5x}} + 2x\cos\left(\sqrt{2-5x}\right)$$

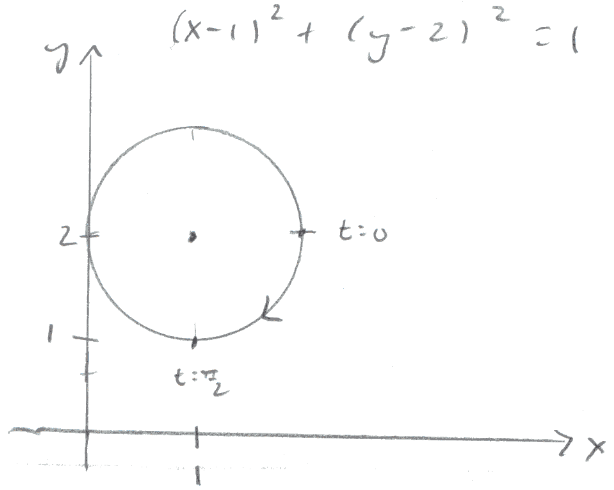
2. The position of a moving particle is described by the parametric equations

$$x = 1 + \cos t, \quad y = 2 - \sin t, \quad 0 \le t \le 2\pi.$$

Find the Cartesian equation that describes the curve along which the particle moves, and please sketch the curve. Be sure to indicate the direction of motion for the particle and state the initial point.

$$\left.\begin{aligned} x-1 &= \cos t \\ y-2 &= -\sin t \end{aligned}\right\} \Rightarrow (x-1)^2 + (y-2)^2 = [\cos t]^2 + [-\sin t]^2 = \cos^2 t + \sin^2 t = 1$$

$(x-1)^2 + (y-2)^2 = 1$, circle with center $(1,2)$ and radius $= 1$



$x(0) = 1 + \cos(0) = 2$
$y(0) = 2 - \sin(0) = 2$
$\Rightarrow (2,2) =$ initial point

To find direction of motion:

$x(\frac{\pi}{2}) = 1 + \cos(\frac{\pi}{2}) = 1$
$y(\frac{\pi}{2}) = 2 - \sin(\frac{\pi}{2}) = 1$

CLOCKWISE

Math 131, Spring 2004
Quiz #6, Discussion Section C (Thursday, 12:00-1:00)

Quiz problems should be solved using the methods discussed in this course. A calculator is not permitted. To receive full credit, show enough work to make it clear how you got your answer.

Name: KEY ID#

Differentiate the following functions, and please simplify your work as much as possible. (2 points each)

1. What is the equation of the tangent line to the curve

$$y^2 e^{6x} + \sin(y-1) = 1$$

at the point $(0, 1)$?

$$\frac{d}{dx}\left(y^2 e^{6x} + \sin(y-1)\right) = \frac{d}{dx}(1)$$

$$y^2(6e^{6x}) + e^{6x}\left(2y\frac{dy}{dx}\right) + \cos(y-1)\frac{dy}{dx} = 0$$

$$2ye^{6x}\frac{dy}{dx} - \cos(y-1)\frac{dy}{dx} = -6y^2e^{6x}$$

$$\frac{dy}{dx}\left[2ye^{6x} - \cos(y-1)\right] = -6y^2e^{6x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-6y^2e^{6x}}{2ye^{6x} - \cos(y-1)}$$

evaluate $\frac{dy}{dx}$ @ $(0,1)$

$$\left.\frac{dy}{dx}\right|_{(0,1)} = \frac{-6(1^2)e^{6(0)}}{2(1)e^{6(0)} - \cos(1-1)} = \frac{-6}{2-1} = -3$$

$$y - 1 = -3(x-0)$$

$$\boxed{y = -3x + 1}$$

2. Suppose a curve is given parametrically by

$$x = 2\cos t, \quad y = 3\sin t, \quad 0 \le t \le 2\pi.$$

What is the **slope** of the tangent line to the curve at the point $t = \frac{\pi}{4}$?

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3\cos t}{-2\sin t}$$

$$\left.\frac{dy}{dx}\right|_{t=\frac{\pi}{4}} = \left.\frac{3\cos t}{-2\sin t}\right|_{t=\frac{\pi}{4}} = \frac{3\cos\left(\frac{\pi}{4}\right)}{-2\sin\left(\frac{\pi}{4}\right)} = \frac{3\frac{\sqrt{2}}{2}}{-2\frac{\sqrt{2}}{2}} = -\frac{3}{2}$$