Math 131, Spring 2004
Quiz #8, Discussion Section A (Thursday, 11:00-12:00)

Quiz problems should be solved using the methods discussed in this course. A calculator is not permitted. To receive full credit, show enough work to make it clear how you got your answer.

Name: Answer Key ID# ____________

1. Suppose $f''(x) = \dfrac{x^2(2x+3)(x-1)}{x^2+2}$. Please find all of the inflection points of $f(x)$ and explain why each is an inflection point.

$f''(x)=0 \iff x=0,\ x=-3/2,\ x=1$; $f''(x)$ is everywhere defined

on $(-\infty,-3/2)$, $f''(x)>0 \Rightarrow f$ concave up

on $(-3/2,0)$, $f''(x)<0 \Rightarrow f$ concave down

on $(0,1)$, $f''(x)<0 \Rightarrow f$ concave down

on $(1,\infty)$, $f''(x)>0 \Rightarrow f$ concave up.

So $x=-3/2$ & $x=1$ are inflection points because at these points f changes concavity

2. For a constant $b>0$, find $\lim_{t\to 0} \dfrac{\sqrt{b-t}-\sqrt{b}}{t}$.

$$\lim_{t\to 0} \frac{\sqrt{b-t}-\sqrt{b}}{t} \overset{LH}{=} \lim_{t\to 0} \frac{\frac{1}{2}(b-t)^{-1/2}(-1)}{1}$$

$$= \lim_{t\to 0} \frac{-1}{2\sqrt{b-t}}$$

$$= -\frac{1}{2\sqrt{b}}$$

Math 131, Spring 2004
Quiz #8, Discussion Section B (Tuesday, 12:00-1:00)

Quiz problems should be solved using the methods discussed in this course. A calculator is not permitted. To receive full credit, show enough work to make it clear how you got your answer.

Name: Answer Key ID# ______

1. Suppose $f'(x) = \dfrac{2x(x-4)}{(x+2)^2}$. Find the critical points of $f(x)$, and please explain why each one is a local maximum, local minimum, or neither.

critical points occur where $2x(x-4)=0$ & $(x+2)^2=0$.
So $x=0$, $x=4$, & $x=-2$ are critical points.

On $(-\infty,-2)$, $f'(x)>0 \Rightarrow f$ increasing
On $(-2,0)$, $f'(x)>0 \Rightarrow f$ increasing
On $(0,4)$, $f'(x)<0 \Rightarrow f$ decreasing
On $(4,\infty)$, $f'(x)>0 \Rightarrow f$ increasing

So $x=0$ is a local max & $x=4$ is a local min.
$x=-2$ is neither.

2. Find the maximum value of $f(x) = \sin x - \cos x$ for $0 \le x \le \pi$.

$f(x) = \sin x - \cos x$

$f'(x) = \cos x + \sin x$

$f'(x) = 0 \iff \cos x = -\sin x \iff x = 3\pi/4$

$f(0) = -1$, $f(3\pi/4) = \sqrt{2}/2 + \sqrt{2}/2 = \sqrt{2}$, $f(\pi) = 1$

So $3\pi/4$ is the max value of f on $0 \le x \le \pi$.

Math 131, Spring 2004
Quiz #8, Discussion Section C (Thursday, 12:00-1:00)

Quiz problems should be solved using the methods discussed in this course. A calculator is not permitted. To receive full credit, show enough work to make it clear how you got your answer.

Name: ______________________ ID# ______________________

1. Suppose $f'(x) = \dfrac{x(x^2-4)}{x^2+1}$. Please find the critical values of $f(x)$, and please explain why each one is a local maximum, local minimum, or neither.

Critical numbers of f occur where $f'(x)=0$ & where $f'(x)$ is undefined, so in this case we have that the critical values of $f(x)$ are $x=0$ & $x=\pm 2$.

On $(-\infty,-2)$, $f'(x)<0 \Rightarrow f$ decreasing
on $(-2,0)$, $f'(x)>0 \Rightarrow f$ increasing
on $(0,2)$, $f'(x)<0 \Rightarrow f$ decreasing
on $(2,\infty)$, $f'(x)>0 \Rightarrow f$ increasing

So f has local maximums at $x=0$ & local minimums at $x=\pm 2$.

2. Evaluate $\displaystyle\lim_{x\to\infty} \frac{\ln(\ln x)}{\ln x}$.

$$\lim_{x\to\infty} \frac{\ln(\ln x)}{\ln x} \overset{LH}{=} \lim_{x\to\infty} \frac{\frac{1}{\ln x}\left(\frac{1}{x}\right)}{\frac{1}{x}}$$

$$= \lim_{x\to\infty} \frac{x}{x\ln x}$$

$$= \lim_{x\to\infty} \frac{1}{\ln x}$$

$$= 0$$