Math 131, Spring 2004
Quiz #9, Discussion Section A (Thursday, 11:00-12:00)

Quiz problems should be solved using the methods discussed in this course. A calculator is not permitted. To receive full credit, show enough work to make it clear how you got your answer.

Name: Answer Key ID# _____

1. Find $f(x)$ if $f'(x) = 1 + 2\sin x - \cos x$ with $f(0) = 3$.

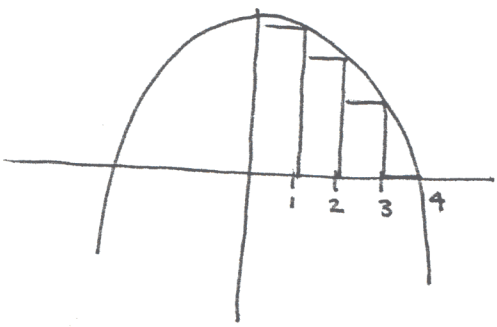
$$f(x) = x - 2\cos x - \sin x + C$$

$$f(0) = 3 \Rightarrow -2 + C = 3$$

$$\Rightarrow C = 5$$

So $f(x) = x - 2\cos x - \sin x + 5$

2. Estimate the area under the graph of $f(x) = 16 - x^2$ from $x = 0$ to $x = 4$ using four approximating rectangles with right endpoints. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate?



$$\Delta x = \frac{b-a}{n} = \frac{4-0}{4} = 1$$

$$R_4 = \sum_{i=1}^{4} f(x_i)\Delta x$$

$$= f(1) + f(2) + f(3) + f(4)$$

$$= 15 + 12 + 7 + 0$$

$$= 34$$

f decreasing from 0 to 4, so R_4 gives an underestimate.

Math 131, Spring 2004
Quiz #9, Discussion Section B (Tuesday, 12:00-1:00)

Quiz problems should be solved using the methods discussed in this course. A calculator is not permitted. To receive full credit, show enough work to make it clear how you got your answer.

Name: Answer Key ID# _____

1. Suppose you use Newton's Method, starting with $x_1 = 1$, to approximate a root of the function $f(x) = x^3 - 2$. What is the value of x_3? You don't have to simplify your final answer (but we would be happy if you did).

$f(x) = x^3 - 2 \Rightarrow f'(x) = 3x^2$

So $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} = x_n - \dfrac{x_n^3 - 2}{3x_n^2}$

$x_1 = 1 \Rightarrow x_2 = 1 - \dfrac{1-2}{3} = 1 + \dfrac{1}{3} = \dfrac{4}{3}$

$x_2 = 4/3 \Rightarrow x_3 = \dfrac{4}{3} - \dfrac{(4/3)^3 - 2}{3(4/3)^2} = \dfrac{4}{3} - 5/72 = 91/72$

2. Find the most general antiderivative of the following functions:

a) (one point) $f(x) = \sec x \tan x$

$F(x) = \sec x + C$

b) (two points) $f(x) = \dfrac{5x^2 + x^{10}}{\sqrt{x}}$.

$f(x) = \dfrac{5x^2 + x^{10}}{\sqrt{x}} = 5x^{3/2} + x^{19/2}$

$F(x) = 5\left(\frac{2}{5}\right)x^{5/2} + \left(\frac{2}{21}\right)x^{21/2} + C$

$= 2x^{5/2} + \left(\frac{2}{21}\right)x^{21/2} + C.$

Math 131, Spring 2004
Quiz #9, Discussion Section C (Thursday, 12:00-1:00)

Quiz problems should be solved using the methods discussed in this course. A calculator is not permitted. To receive full credit, show enough work to make it clear how you got your answer.

Name: _____ ID# _____

1. Find the most general antiderivative of the following functions:

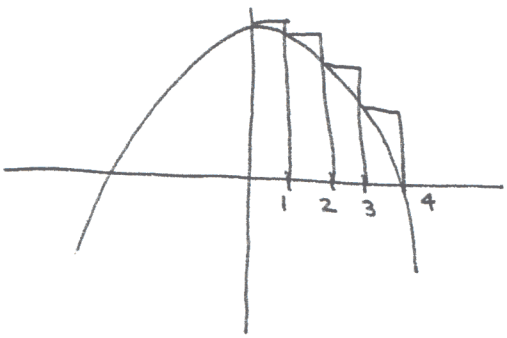
a) (one point) $f(x) = \sec^2 x$

$F(x) = \tan x + C$

b) (two points) $\frac{x^2 - 2\sqrt{x}}{x} = x - 2x^{-1/2}$

$F(x) = \frac{x^2}{2} - 4x^{1/2} + C \qquad x > 0$

2. Estimate the area under the graph of $f(x) = 16 - x^2$ from $x = 0$ to $x = 4$ using four approximating rectangles with left endpoints. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate?



$\Delta x = \frac{b-a}{n} = \frac{4-0}{4} = 1$

$L_4 = \sum_{i=1}^{4} f(x_{i-1})\Delta x$

$= f(0) + f(1) + f(2) + f(3)$

$= 16 + 15 + 12 + 7$

$= 50$

f decreasing from 0 to 4, so L4 gives an overestimate.