MATH 233 (SPRING 2016)
MIDTERM EXAM 2 REVIEW SUGGESTIONS

This two-hour exam will consist of 20 multiple-choice questions, covering material through Lecture 20. Make sure to bring a number 2 pencil. Calculators and 3×5-cards will not be permitted. Exam scores will be available the following day.

The exam questions are drawn from the following list of skills and topics from §§13.3-4 and 14.1-7 in Stewart. Topics not listed (tangential and normal components of acceleration, Kepler's laws, etc.) will not be covered.

13.3	Finding arclength by integrating speed: $s = \int_a^b \|\|\vec{r}'(t)\|\| dt$, or if you want it as a function of t, $s(t) = \int_0^t \|\|\vec{r}'(u)\|\| du$.
	Reparametrizing $\vec{r}(t)$ by arclength: by computing $s(t)$ then its inverse $t(s)$, and plugging this into $\vec{r}(t)$.
	Computing curvature $\kappa(t)$: know formulas 9 and 10 from the book, and what κ means geometrically.
	Finding $\vec{N}$ and $\vec{B}$, and using $\vec{B}$ to write the equation of the osculating plane.
13.4	Given $\vec{a}(t)$, $\vec{v}(0)$, and $\vec{r}(0)$, finding $\vec{r}(t)$ by integrating twice.
	Physics type problems, involving gravity (like the ballistics problems we did).
13.5	There is no 13.5, relax.
14.1	Sketching/recognizing $z = f(x, y)$; drawing contour maps (level curves) and domain of f. Identifying where a function is continuous.
14.2	Finding limits (one approach = convert to polar), or showing they don't exist (e.g. by considering different paths).
14.3	Computing partial derivatives $f_x = \frac{\partial f}{\partial x}$, higher partials; know $f_{xy} = f_{yx}$ always (assuming they're continuous).
14.4	Equation for tangent plane to $z = f(x, y)$ at (x_0, y_0), given by $z = L(x, y)$. This doubles as "linear approximation".
	Using $L(x, y)$ to estimate value of f at a point near (x_0, y_0).
	Using differentials to estimate error and so forth.
14.5	Chain rule. Don't memorize formulas for special cases; instead, understand how to draw the diagrams and use them to produce the formula.
14.6	Gradient vector $\vec{\nabla} f = \langle f_x, f_y \rangle$ ($\langle f_x, f_y, f_z \rangle$ for functions of 3 variables) and its properties: normal to level curves, points in direction of steepest increase.
	Use this to find formulas for tangent lines (planes) to level curves (surfaces).
	Directional derivative $D_{\hat{u}} f = (\vec{\nabla} f) \cdot \hat{u}$ and its intuitive meaning.
14.7	Find *local* extrema and saddle points of a function on xy-plane, using the second derivative test.
	Optimization: finding *global* extrema of functions arising from practical or geometric problems. (Usually this is done on a closed bounded subset of $\mathbb{R}^2$, but sometimes the set isn't bounded.)

Practice Midterm 2

Part I: Multiple choice problems

(1) If your Prius XLVII spaceship had velocity vector $\vec{v}(t) = \langle 4, 3\cos t, 3\sin t\rangle$ from $t = 0$ to $t = \pi$, compute the length of the arc along which you traveled.
(A) π
(B) 2π
(C) 3π
(D) 4π
(E) $\sqrt{4\pi^2+9}$
(F) $2\sqrt{4\pi^2+9}$
(G) none of the above

(2) To reparametrize $\vec{r}(t) = \langle e^t \sin t, e^t \cos t, e^t\rangle$ by arclength from $t = 0$, you must replace t by $t(s) =$ ____.
(A) $\sqrt{3}(e^s - 1)$
(B) $\sqrt{3}e^s$
(C) $\ln(s+1)$
(D) $\ln(\frac{s}{\sqrt{2}}+1)$
(E) $\ln(\frac{s}{\sqrt{3}}+1)$
(F) $\ln(\frac{s}{\sqrt{3}})$
(G) none of the above

(3) Find $\vec{r}(t)$ if $\vec{a}(t) = \langle 0, 0, 1\rangle$, $\vec{v}(0) = \langle 1, -1, 0\rangle$, and $\vec{r}(0) = \langle 0, 1, 0\rangle$.
(A) $\langle 0, 1, \frac{t^2}{2}\rangle$
(B) $\langle t, 1-t, \frac{t^2}{2}\rangle$
(C) $\langle t, -t, \frac{t^2}{2}\rangle$
(D) $\langle 1, -1, t\rangle$
(E) $\langle t, 1-t, t^2\rangle$
(F) none of the above

(4) Let $\vec{r}(t) = \langle 1 + \frac{t^4}{4}, \frac{\sqrt{2}t^3}{3}, \frac{t^2}{2}\rangle$. What is the radius of the osculating circle at $\vec{r}(\sqrt{2})$? [Warning: this is a bit of a long computation. You may wish to leave this and the next problem for last.]
(A) 9
(B) 3
(C) 1
(D) $\frac{1}{3}$
(E) $\frac{1}{9}$
(F) none of the above

(5) With $\vec{r}(t)$ as in problem (4), which is an equation of the osculating plane at $t = \sqrt{2}$?
(A) $6x + 6y + 3z = 26$
(B) $-6x + 3y + 6z = 4$
(C) $3x - 6y + 6z = 26$
(D) $3x - 6y + 6z = 10$
(E) $-6x + 3y + 6z = 0$
(F) $6x + 6y + 3z = 6$
(G) none of the above

(6) A bee was flying along a helical path $\vec{H}(t) = \langle \cos t, \sin t, 16t \rangle$ (measured in feet). At $t = 12$, it had a heart attack and died instantly. Where did it land (that is, what are the (x, y)-coordinates where it hit the xy-plane)? [Hint: reset $t = 0$ when the bee dies, and use the acceleration due to gravity $g = 32$ ft/sec^2.]
(A) $(\cos 12, \sin 12)$
(B) $(-4\sin 12, 4\cos 12)$
(C) $(\cos 12 + 3\sin 12, \sin 12 - 3\cos 12)$
(D) $(\cos 12 - \sin 12, \sin 12 + \cos 12)$
(E) $(\cos 12 - 4\sin 12, \sin 12 + 4\cos 12)$
(F) none of the above

(7) What is $\lim_{(x,y)\to(0,0)} \frac{\sin(3x^2+3y^2)}{x^2+y^2}$?
(A) 1
(B) $\frac{1}{3}$
(C) 3
(D) 9
(E) 0
(F) DNE
(G) none of the above

(8) Use the linear approximation to $f(x, y) = x^2/y$ at $(1, 1)$ to approximate $0.99^2/1.01$.
(A) 1
(B) 0.99
(C) 0.98
(D) 0.97
(E) 0.96
(F) 0.95
(G) none of the above

(9) Which is an equation of the plane tangent to the surface $z + xe^{-2y} = y^2e^z + 2$ at $(1, -1, 2)$?
(A) $e^2x + (1 - e^2)z = 2 - e^2$
(B) $e^2x + e^2y + (1 - e^2)z = 2 - 2e^2$
(C) $e^2y + (1 - e^2)z = 2 - 3e^2$
(D) $-e^2x + (1 - e^2)z = 2 - 3e^2$
(E) $e^2x + (1 - e^2)z = e^2 - 2$
(F) $e^2x + e^2y - e^2z = -2e^2$
(G) none of the above

(10) With $f(x, y) = y^2 \ln x$, find the slope of the graph of $z = f(x, y)$ in the direction $\vec{v} = \langle -1, 1 \rangle$, "over" the point $p = (1, 2)$.
(A) -4
(B) -2
(C) 0
(D) 2
(E) 4
(F) none of the above

(11) With $f(x, y)$ as in problem (10), the direction of steepest slope of the graph at $(1, 2)$ makes what angle with the vector $\langle 1, 0\rangle$?
(A) $0°$
(B) $45°$
(C) $90°$
(D) $135°$
(E) $180°$
(F) none of the above

(12) Determine the dimensions of a rectangular box without a lid, of volume 4 m^3, which minimize the surface area (hence material used). What is the height?
(A) $\frac{1}{\sqrt[3]{4}}$
(B) $\frac{1}{\sqrt[3]{2}}$
(C) 1
(D) $\sqrt[3]{2}$
(E) $\sqrt[3]{4}$
(F) 2
(G) 4
(H) none of the above

Part II: Hand-graded problems

(1) Find all critical points of $f(x, y) = x^3 + y^3 - 6xy$ on the xy-plane. Use the second-partials test to decide whether each point gives a local maximum, minimum, or saddle point.
(2) If $T = f(x, y, z, w)$, and x, y, z, w are each functions of s and t, write a chain rule for $\partial T/\partial s$. Draw a diagram if you like.
(3) Compute (or show does not exist): $\lim_{(x,y)\to(0,0)} \frac{xy+y^3}{x^2+y^2}$.
(4) Sketch (a) the graph of $f(x, y) = e^{-(x^2+y^2)}$, (b) a rough contour map (level curves) for $f(x, y) = x^2 + y$, and (c) the largest set on which $f(x, y) = \ln(1 - x^2 - y^2)$ is continuous.
(5) I want a function $f(x, y)$ with $f_x(x, y) = \cos(x) - \sin(y)$ and $f_y(x, y) = \cos(y) + \sin(x)$, but I'm having trouble finding one. What gives?