# MATH 233 (SPRING 2016)
# FINAL EXAM REVIEW SUGGESTIONS

This two-hour exam will consist of 25 multiple-choice questions, 10 of which are on the new material (sections 16.2-6). (Don't panic: the questions are mostly easier than on previous exams, so while more numerous should go by more quickly.) Make sure to bring a number 2 pencil. Calculators and $3 \times 5$-cards will not be permitted. Exam scores will be available the following day.

Below is a list of topics from Chapter 16. For the earlier material, refer to previous exam study guides, with the following caveats: you do not need to know the point-plane distance formula, or how to parametrize the osculating circle.

16.1 Idea of a vector field $\vec{F}(x, y, z)$ (or $\vec{F}(x, y)$), flow lines (parametric curves solving $\vec{r}'(t) = \vec{F}(\vec{r}(t))$).
Definition: $\vec{F}$ is *conservative* if $\vec{F} = \vec{\nabla} f$ for some function $f$

16.2 How to compute line integrals with respect to arclength ($ds$), $dx$, and $dy$. (Replace these, respectively, by $\|\vec{r}'(t)\|\, dt$, $x'(t)dt$, and $y'(t)dt$.)
If $\vec{F}(x, y) = P(x, y)\mathbf{i} + Q(x, y)\mathbf{j}$, then $\int_C \vec{F} \cdot d\vec{r} = \int_C P dx + Q dy = \int_a^b \{P(x(t), y(t))x'(t) + Q(x(t), y(t))y'(t)\}dt$.

16.3 $\int_C \vec{F} \cdot d\vec{r}$ independent of path $\iff$ $\oint_C \vec{F} \cdot d\vec{r} = 0$ (for all loops) $\iff$ $\vec{F}$ conservative.
On a simply connected region, $\vec{F}$ conservative $\iff$ curl$\vec{F} = \vec{0}$, i.e. $P_y = Q_x$ (and $P_z = R_x$, $Q_z = R_y$ if in 3-D).
In this case, be able to find $f$ such that $\vec{F} = \vec{\nabla} f$, and use to compute $\int_C \vec{F} \cdot d\vec{r} = f(B) - f(A)$.

16.4 Green's theorem (in plane) $\oint_{\partial D} P dx + Q dy = \iint_D (Q_x - P_y) dx\, dy$.
Application to computing area: $A(D) = \iint_D dx\, dy = \oint_C x\, dy = \frac{1}{2} \oint (-y\, dx + x\, dy)$.

16.5 $\text{curl}(\vec{F}) = \langle R_y - Q_z, P_z - R_x, Q_x - P_y \rangle$, $\text{div}(\vec{F}) = P_x + Q_y + R_z$, $\nabla^2 f = f_{xx} + f_{yy} + f_{zz}$
On $\mathbb{R}^3$, $\vec{F}$ is the curl of another field $\iff$ $\text{div}(\vec{F}) = 0$.
Gauss's theorem (in plane): $\oint_{\partial D} \vec{F} \cdot \hat{n} ds = \iint_D \text{div}(\vec{F}) dA$.

16.6 Surface area of a parametric surface $S$ parametrized by $\vec{r}(u, v)$ (where $(u, v)$ range over $D \subset \mathbb{R}^2$): given by $\iint_D \|\vec{r}_u \times \vec{r}_v\|\, du\, dv$, or in case of graph $z = f(x, y)$ by $\iint_D \sqrt{1 + (f_x)^2 + (f_y)^2} dx\, dy$.