MATH 233 (SPRING 2016)
MIDTERM EXAM 1 REVIEW SUGGESTIONS

This two-hour exam will consist of 20 multiple-choice questions, covering material through Lecture 8. Make sure to bring a number 2 pencil. Calculators and 3×5-cards will not be permitted. Exam scores will be available the following morning.

The exam questions are drawn from the following list of skills and topics. Topics not listed (e.g. torque, foci of a conic, etc.) will not be covered. Besides the formula for the distance from a point to a plane, another formula you may find useful on the exam is $\sin(2\theta) = 2\cos\theta\sin\theta$.

Chapter 10 (plane curves):

- Eliminate the parameter to go from parametric equations to "Cartesian" (equation in x and y), and sketch easy examples.
- Convert between polar and Cartesian coordinates and equations, particularly for equations describing conics.
- Be able to recognize equations of conics, and distinguish hyperbolas, parabolas, ellipses.

Chapter 12 (vectors, lines, planes, quadrics):

- Compute $\vec{a} \times \vec{b}$, $\vec{a} \cdot \vec{b}$, and scalar triple product; know $\vec{a} \cdot \vec{b} = \|\vec{a}\| \left\|\vec{b}\right\| \cos\theta$, $\left\|\vec{a} \times \vec{b}\right\| = \|\vec{a}\| \left\|\vec{b}\right\| |\sin\theta|$; unit vectors.
- Find vector and scalar projections (e.g. "$\vec{v}$ onto $\vec{w}$"), know $\vec{a} \perp \vec{b} \iff \vec{a} \cdot \vec{b} = 0$; compute work.
- Find area of parallelogram or triangle spanned by 2 vectors; volume of parallelepiped spanned by 3 vectors.
- Know distance formula, equations of planes and quadrics (including cylinders and spheres, and completing the square to put in standard form); recognize surface given in terms of distance data.
- Know parametric and symmetric equations of lines. Find lines' equations from: point/vector on line; intersection of 2 quadrics; or through point and parallel or perpendicular to another line; or through two points.
- Find planes' equations from: point and normal vector; 3 points on plane; point and line in plane; parallel to another plane.
- Find distance between: point and plane (know formula $\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$); point and line; plane and line; 2 lines (skew)
- Find angle between 2 planes, 2 vectors, line and plane.
- Tension in clothesline and related problems (where force vectors must add to $\vec{0}$).

Chapter 13 (vector-valued functions – up to §13.2):

- Limits and derivatives of vector functions.
- Find equations satisfied by the image curve, sketch image curve (both in simple situations).
- Unit tangent vector and smoothness/non-smoothness of the image curve.
- Parametric equation for the tangent line to a parametrized curve: formula $\vec{\ell}_a(u) = \vec{r}(a) + u\vec{r}'(a)$.
- Angle made by 2 curves meeting at a point (i.e. the angle made by their "velocity vectors" there).

Practice Midterm 1

Part I: Multiple choice problems

Note: the actual exam won't have "none of the above" options on every problem like this one does.

(1) Find the area of the triangle with vertices $(0, 0, 0)$, $(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3})$, and $(1, 2, 2)$.
(A) $\frac{1}{2}$
(B) 1
(C) $\frac{3}{2}$
(D) 2
(E) $\frac{5}{2}$
(F) 3
(G) $\frac{7}{2}$
(H) 4
(I) none of the above

(2) Which plane is parallel to the one containing the triangle of problem (1)?
(A) $2(x-1)+(y-2)+2(z-2)=1$
(B) $-2x+y+2z=-1$
(C) $-4x+y+z=3$
(D) $(x-\frac{2}{3})+2(y+\frac{2}{3})+2(z-\frac{2}{3})=0$
(E) $2x+y-2z=5$
(F) none of the above

(3) Find the angle between the planes $x+y+2z=4$ and $x+z=-2$.
(A) $30°$
(B) $45°$
(C) $60°$
(D) $90°$
(E) parallel
(F) $-20°F$
(G) none of the above

(4) What are equations for the line at the intersection of the two planes in problem (3)?
(A) $x=-1-t,\ y=-t,\ z=-1+t$
(B) $x=t,\ y=8-t,\ z=-2-t$
(C) $x=-2+t,\ y=6+t,\ z=-t$
(D) $x=-8-t,\ y=t,\ z=6+t$
(E) all of the above
(F) none of the above

(5) Determine the distance between the point $(4, -2, 3)$ and the plane $4x-4y+2z=2$.
(A) 1
(B) 2
(C) 5
(D) $\frac{10}{3}$
(E) $\frac{14}{3}$
(F) $\frac{16}{3}$
(G) none of the above

(6) Which is an equation for the plane parallel to the plane of problem (5) and through $(4, -2, 3)$?
(A) $4x - 4y + 2z = -15$
(B) $-2x + 2y - z = -15$
(C) $4x - 4y + 2z = 14$
(D) $4x - 2y + 3z = 30$
(E) all of the above
(F) none of the above

(7) Find the equation (in x, y, z) of the cone on which $\vec{r}(t) = \langle t \sin t,\ t \cos t,\ \sqrt{8}t \rangle$ lies. Which point lies on this cone?
(A) $(3, -4, -10\sqrt{2})$
(B) $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, \sqrt{8})$
(C) $(\sqrt{2}, 0, -4)$
(D) $(-\sqrt{2}, 4, 12)$
(E) all of the above
(F) none of the above

(8) Identify $r = \frac{6}{2+\sin\theta}$ as a conic.
(A) hyperbola centered at $(0, 2)$
(B) parabola with vertex $(0, -2)$
(C) ellipse with center $(0, -2)$
(D) parabola with vertex $(2, 0)$
(E) circle with center $(0, -2)$
(F) ellipse with center $(2, 0)$
(G) asymptotic hypotenuse
(H) polar bear
(I) none of the above

(9) A 100 N weight sits on a 30° incline. How much force, applied parallel to the surface of the incline, will prevent it sliding down?
(A) 0 N
(B) 50 N
(C) $50\sqrt{3}$ N
(D) 100 N
(E) $100\sqrt{3}$ N
(F) 200 N
(G) none of the above

(10) Find the center and radius of $x^2 + y^2 + z^2 - 4x + 10y - 8z = 36$.
(A) center $(4, -10, 8)$, radius 6
(B) center $(2, -5, 4)$, radius 9
(C) center $(-4, 10, -8)$, radius 3
(D) center $(-2, 5, -4)$, radius 6
(E) center $(4, -10, 8)$, radius 9
(F) center $(0, 0, 0)$, radius 3
(G) center $(2, -5, 4)$, radius 6
(H) none of the above

(11) Consider a box with dimensions $1ft \times 1ft \times 4ft$. Let θ be the angle between the diagonal of the box and the diagonal of the $1ft \times 1ft$ side. What is $\cos\theta$?
(A) 0
(B) $\frac{1}{6}$
(C) $\frac{\sqrt{3}}{2}$
(D) $\frac{1}{4}$
(E) $\frac{1}{3}$
(F) $\frac{1}{2}$
(G) $\frac{2}{\sqrt{3}}$
(H) $-\frac{1}{6}$
(I) none of the above

(12) Calculate the vector projection of $\vec{b} = \langle -3, 2\rangle$ onto $\vec{a} = \langle 3, 4\rangle$.
(A) $\langle -\frac{3}{25}, -\frac{4}{25}\rangle$
(B) $\langle \frac{3}{5}, \frac{4}{5}\rangle$
(C) $\langle -3, -4\rangle$
(D) $-\frac{1}{5}$
(E) $\langle \frac{3}{5}, \frac{4}{5}\rangle$
(F) $\frac{1}{5}$
(G) $\langle 3, 4\rangle$
(H) none of the above

(13) Which are equations of the line through $(0, 3, 8)$ and $(-1, 4, 6)$?
(A) $x = -t$, $y = 3 + 4t$, $z = 8 + 6t$
(B) $x = -2 + 3t$, $y = 5 - 3t$, $z = 4 + 6t$
(C) $x - 1 = 4 - y = \frac{z-6}{2}$
(D) $x = y - 3 = \frac{z-8}{2}$
(E) $x = -1 + 2t$, $y = 4 - 2t$, $z = 6 + 2t$
(F) all of the above
(G) none of the above

(14) Which expression is meaningless?
(A) $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$
(B) $(\vec{a} \times \vec{b}) \times \vec{c}$
(C) $(\vec{a} \cdot \vec{b})\vec{c} - (\vec{a} \cdot \vec{c})\vec{b}$
(D) $\vec{a} \times (\vec{b} \cdot \vec{c})$
(E) $\vec{a} \cdot (\vec{b} \times \vec{c})$
(F) none of the above
(G) all of the above
(H) all mathematical expressions

(15) Find the distance between the lines $\frac{x-3}{2} = \frac{y+2}{-2} = z - 1$ and $x + 4 = \frac{y+5}{2} = \frac{z}{2}$.
(A) zero: they intersect
(B) 1
(C) 2
(D) 3
(E) 4
(F) 5
(G) 6
(H) none of the above

Part II: Free response problems

(1) You think you're about to take a calculus test in Colorado, but then you notice you're on a roller-coaster ride in the intergalactic space station. Suddenly you decide you need to go to the intergalactic lavatory (located for your convenience at $(-4, 0, 7)$). Since there's no gravity, at the instant you release your seat-belt you will fly off in the tangent direction. Let's figure out how to time this.

(a) Write an abstract formula for the tangent line to $\vec{r}(t)$ [= the roller-coaster] at $t = a$. Let me suggest using some variable besides t (say u) to parametrize the tangent line, as in $\vec{\ell}_a(u)$.

(b) Let's say $\vec{r}(t) = \langle 2\cos t,\, 3\sin t,\, \cos 5t\rangle$. Draw a very rough sketch of the curve this traces out, and find $\vec{\ell}_a(u)$ by plugging in your answer to part (a).

(c) Solve for a value of a which makes the tangent line $\vec{\ell}_a(u)$ pass through the lavatory. (This is when you should self-eject.)

(2) (a) Find the equation of the surface consisting of all points equidistant from the xz-plane and the point $(0, 2, 0)$.

(b) Identify and sketch the surface.

(3) Let $\vec{r}(t) = \langle 1 + \frac{t^4}{4}, \frac{\sqrt{2}t^3}{3}, \frac{t^2}{2} \rangle$. Find the unit tangent vector $\vec{T}(t)$ and use this to decide whether the space curve traced out by $\vec{r}(t)$ is everywhere smooth. [Hint/Warning: you really do need $\vec{T}(t)$.]