MATH 233 LECTURE 11 (§13.4): ACCELERATION

Physics of curvilinear motion.

- If $\vec{r}(t)$ describes the motion of an object, then $\vec{v}(t) := \vec{r}'(t)$ is its velocity, and $\vec{a}(t) := \vec{r}''(t)$ is its *acceleration*. We will also write $\nu(t) = \|\vec{r}'(t)\|$ for speed.
- If the object has mass m, then by Newton's 2nd law $\vec{F}(t) = m\vec{a}(t)$, where $\vec{F}(t)$ is the (vector) sum of all the forces acting on the object at time t.
- In uniform circular motion, $\vec{a}(t)$ points toward the center of the circle.
- For projectile motion in the xy-plane (thinking of the y-direction as "up"), we assume $\vec{a}(t) = \vec{a} = \langle 0, -g\rangle$ ($g = 32\ ft/s^2$ or $9.8m/s^2$). Choose an initial velocity vector $\vec{v}(0) = \langle \nu_0 \cos\theta, \nu_0 \sin\theta\rangle$ and position vector $\vec{r}(0) = \langle x_0, y_0\rangle$. Use these as constants of integration to get $\vec{v}(t) = \langle \nu_0 \cos\theta, -gt + \nu_0 \sin\theta\rangle$ and $\vec{r}(t) = \langle (\nu_0 \cos\theta)t + x_0, -\frac{g}{2}t^2 + (\nu_0 \sin\theta)t + y_0\rangle$.
- More generally (for motion in 2 or 3 dimensions), you should be able to obtain $\vec{r}(t)$ from $\vec{a}(t)$ (or $\vec{F}(t)$), $\vec{v}(0)$ and $\vec{r}(0)$.
- Tangential (speed-changing) and normal (direction-changing) components of acceleration: $\vec{a}(t) = a_T(t)\vec{T}(t) + a_N(t)\vec{N}(t) = \nu'(t)\vec{T}(t) + (\nu(t))^2\kappa(t)\vec{N}(t)$.

Kepler's laws of planetary motion.

- One of Newton's great accomplishments with his calculus was the proof that Kepler's laws follow from Newton's laws of motion and gravitation.
- 1st law: the motion is elliptical with the sun as a focus. (Denote the length of the semi-major axis by a.)
- 2nd law: area is swept out at a constant rate (here area refers to the "piece of pie" with vertex at the sun and outer edge given by the planet's motion). Will prove this one in class.

- 3rd law: the period of revolution (time to go once around the sun) is proportional to $a^{3/2}$.