MATH 233 LECTURE 4 (§12.5, CONT'D):
LINES AND PLANES IN SPACE:
INTERSECTIONS, ANGLES, DISTANCE

Intersections.

- Two distinct planes: given $\mathbb{P}_1$ and $\mathbb{P}_2$, with normal vectors $\vec{n}_1$ and $\vec{n}_2$, there are two possibilities. Either $\mathbb{P}_1$ and $\mathbb{P}_2$ are parallel ($\vec{n}_1$ is a scalar multiple of $\vec{n}_2$), or they intersect in a line ℓ. In the latter case, the direction vector $\vec{v}$ of ℓ is given by $\vec{n}_1 \times \vec{n}_2$. By finding a point P in the intersection, you can then obtain an equation of ℓ.
- Line and plane: if the direction vector of ℓ is not "in" the plane $\mathbb{P}$ (i.e. $\ell \cdot \vec{n} \neq 0$), then ℓ intersects $\mathbb{P}$ in a point. You can find this point by using the parametric equations of ℓ, and solving for the value of t for which $x(t), y(t), z(t)$ satisfy the equation of $\mathbb{P}$.
- Two distinct lines: given ℓ_1 and ℓ_2, they may or may not intersect. You may determine this by writing the parametric equations of ℓ_1 [resp. ℓ_2] in terms of t [resp. s], then trying to solve for s and t such that the x, y, z values coincide (3 equations in 2 variables). If they do not intersect, and they are not parallel (parallel means that the direction vector $\vec{v}_1$ is a scalar multiple of $\vec{v}_2$), then ℓ_1 and ℓ_2 are called *skew* lines.

Angles.

- Two planes: the angle between $\mathbb{P}_1$ and $\mathbb{P}_2$ is just the angle between $\vec{n}_1$ and $\vec{n}_2$.
- Line and plane: find the (acute) angle α between $\vec{v}$ (direction vector of ℓ) and $\vec{n}$ (normal vector of $\mathbb{P}$). Then the angle between ℓ and $\mathbb{P}$ is $\theta = 90^\circ - \alpha$.
- Two intersecting lines: this is easy.

Distance.

- Point to plane: the distance from $Q(x_1, y_1, z_1)$ to the plane $\mathbb{P}$ with equation $ax + by + cz + d = 0$ is given by

$$d(Q, \mathbb{P}) = \text{comp}_{\vec{n}} \overrightarrow{PQ} = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}},$$

  where P is any point on $\mathbb{P}$.

- Point to line: see your homework.
- Two parallel planes or skew lines: see next lecture.