MATH 233 LECTURE 40: THE REST OF VECTOR CALCULUS

Since this material is not on the exam, I just give the briefest of summaries here:

- Recall that the surface integral of a function $g(x, y, z)$ over a surface $S \subset \mathbb{R}^3$ parametrized by $\vec{r}: D \to \mathbb{R}^3$ (where $D \subset \mathbb{R}^2$) is

$$\iint_S g\, dS := \iint_D g(\vec{r}(u, v))\, \|\vec{r}_u \times \vec{r}_v\|\, dA.$$

- As for curves, we have a notion of the flux of a vector field $\vec{F}(x, y, z)$ across S. This is given by

$$\iint_S \vec{F} \cdot \hat{n} dS := \iint_D \vec{F}(\vec{r}(u, v)) \cdot \frac{\vec{r}_u \times \vec{r}_v}{\|\vec{r}_u \times \vec{r}_v\|}\, \|\vec{r}_u \times \vec{r}_v\|\, dA = \iint_D \vec{F}(\vec{r}(u, v)) \cdot (\vec{r}_u \times \vec{r}_v) dA.$$

- Gauss's Divergence Theorem then says that for a solid region $V \subset \mathbb{R}^3$ with boundary $S = \partial V$,

$$\iint_S \vec{F} \cdot \hat{n} dS = \iiint_V \operatorname{div}(\vec{F}) dV.$$

  (We haven't done triple integrals in the course, but they are also computed by iterated integrals and are completely analogous to the 2-D integrals we have done.)

- Stokes's Theorem says that for a parametric surface S with boundary $C = \partial S$,

$$\oint_C \vec{F} \cdot d\vec{r} = \iint_S \operatorname{curl}(\vec{F}) \cdot \hat{n} dS.$$

  Both this and Gauss are generalizations of the Fundamental Theorem of Calculus to higher dimensions.