Problem Set 10

(1) Find polynomials (for each n) realizing the upper bound (namely, $\frac{n}{n+1}$) on $\sigma(P)$ in the Smale Conjecture. (See the end of Lecture 30.)

(2) Examine the proof of Bloch's theorem (Lecture 30) to prove that $L \geq \frac{1}{24}$.

(3) Prove that the power-series coefficients a_n, viewed as functions on the set $\mathcal{S}$ of schlicht functions, are continuous functions in the normal topology. (This implies that there must exist universal upper bounds on them, as $\mathcal{S}$ is compact in this topology.)

(4) Let $\Omega \subsetneq \mathbb{C}$ be a simply connected domain, $\alpha \in \Omega$ a point. The Riemann Mapping Theorem guarantees a conformal isomorphism $F\colon \Omega \to D_1$ with $F(\alpha) = 0$. Show that $\frac{1}{4 \cdot d(\alpha, \partial\Omega)} \leq |F'(\alpha)| \leq \frac{1}{d(\alpha, \partial\Omega)}$. (You'll need to use Köbe $\frac{1}{4}$, which will appear in Monday's notes. Also Schwarz lemma.)