Problem Set 2

(1) Show that the mappings of a disk onto a parallel strip, or onto a half strip with two right angles, can be obtained as special cases of the Schwarz-Christoffel formula. [You may want to think about Exercises 1 and 2 on p. 238 of Alhfors, which come before this, first.]

(2) Show that $F(w) = \int_0^w (1-w^n)^{-2/n} dw$ maps $|w| < 1$ onto the interior of a regular polygon with n sides.

(3) Read the section on triangle functions in Ahlfors. In each of the three cases, determine the configuration of triangles in a parallelogram spanned by the periods. See if you can determine what is special about the torus $\mathbb{C}/\Lambda$ (Λ the period lattice) in these cases. [Hint: automorphisms.]

(4) Let $w = u + iv = f(z) = z + \log z$ for $z \in \mathfrak{H}$. Prove that f gives an isomorphism of $\mathfrak{H}$ with the open set $\mathcal{U}$ obtained from $\mathfrak{H}$ by deleting the half-line of numbers $u + i\pi$ with $u \leq -1$. [Hint: use Theorem B (Lect. 4) applied to the path consisting of the segment from R to ϵ, the small semicircle in $\mathfrak{H}$ from ϵ to $-\epsilon$, the segment from $-\epsilon$ to $-R$, and the large semicircle in $\mathfrak{H}$ from $-R$ to R. Note that if we write $z = re^{i\theta}$, then $f(z) = re^{i\theta} + \log r + i\theta$.]