Problem Set 2

In addition to these problems, read the introduction to Chapter 4 in Jacobson.

(1) [Jacobson p. 215 #2] Determine $[\mathbb{Q}(\sqrt{2}, \sqrt{3}):\mathbb{Q}]$.

(2) [Jacobson p. 215 #4] Let $w = \cos(\pi/6) + \mathbf{i}\sin(\pi/6)$ (in $\mathbb{C}$). Note that $w^{12} = 1$ but $w^r \neq 1$ if $1 \leq r < 12$ (so w is a generator of the cyclic group of 12th roots of 1). Show that $[\mathbb{Q}(w):\mathbb{Q}] = 4$ and determine the minimal polynomial of w over $\mathbb{Q}$.

(3) [Jacobson p. 215 #6] Let E_i, $i = 1, 2$, be a subfield of K/F such that $[E_i:F] < \infty$. Show that if E is the subfield of K generated by E_1 and E_2 then $[E:F] \leq [E_1:F][E_2:F]$.

(4) [Jacobson p. 215 #8] Let $E = F(u)$, u transcendental, and let $K \neq F$ be a subfield of E/F. Show that u is algebraic over K.

(5) Given that $(\ell, 0)$ is constructible ($\ell \in \mathbb{R}_+$), show how to construct $(\sqrt{\ell}, 0)$ and $(\ell^2, 0)$. (You must give more detail than Jacobson.)

(6) Construct a regular pentagon "with straightedge and compass".

(7) Suppose that M/L and L/K are extensions, and that $\alpha \in M$ is algebraic over K. Does $[L(\alpha) : L]$ always divide $[K(\alpha) : K]$?