Math 131, Final Exam, December 13th

This exam should have 20 multiple choice questions. Each multiple-choice question is worth 5 points.

Mark your ID number on the six blank lines on the top of your answer card, using one line for each digit. Print your name on the top of the card.

Choose the answer that is *closest* to the solution — the exact solution may not be on the list. Mark your answer card with a **PENCIL** by shading in the correct box.

You may use a calculator, but not one that has a graphing function, or does symbolic differentiation. Remember that all angles are assumed to be in **RADIANS**. Set your calculator to radians now!

You may not have any written aids.

1. What is the global maximum of $f(x) = 2(1-x)^3$ on the interval $[0, 2]$? (Note: The question is what the maximum value attained by f is, not where it is located).

A. -1.5
B. -1.0
C. -0.5
D. 0.0
E. 0.5
F. 1.0
G. 1.5
H. 2.0
I. 2.5
J. 3.0

$f'(x) = -6(1-x)^2 \leq 0$

So f is decreasing

$\therefore$ Max at 0

$f(0) = 2$

2. What is the slope of the line tangent to the curve given by $x^3 + 3x^2y^2 + 5y^3 + y = 8$ at the point $(2, 0)$?

A. 4
B. -4
C. 8
D. -8
E. 10
F. -10
G. 12
H. -12
I. 14
J. -14

$3x^2 + 6xy^2 + 6x^2y\frac{dy}{dx}$

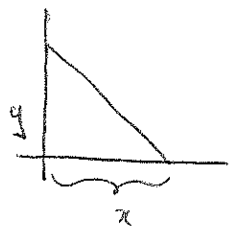
$+ 15y^2\frac{dy}{dx} + \frac{dy}{dx} = 0$

At $(2,0)$:

$12 + \frac{dy}{dx} = 0$

3. A ladder of length 13 feet is leaning against the side of a house and the bottom of the ladder is being pulled away from the house at a rate of 6 feet/second. Assuming that the top of the laddder stays in contact with the house, at what rate is the height of the top of the ladder decreasing when the bottom of the ladder is 5 feet away from the house?

A. 1.50 feet/second
B. 1.75 feet/second
C. 2.00 feet/second
D. 2.25 feet/second
E. 2.50 feet/second
F. 2.75 feet/second
G. 3.00 feet/second
H. 3.25 feet/second
I. 3.50 feet/second
J. 3.75 feet/second



$x^2 + y^2 = 13^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\frac{dx}{dt} = 6 \qquad \frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$

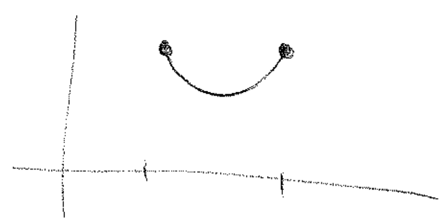
When $x = 5, y = 12$, $\frac{dy}{dt} = -\frac{5}{12} \cdot 6 = -2.5$

4. How many local maxima does

$$f(x) = \frac{1}{\sin(x)}$$

have on the interval $[\frac{\pi}{4}, \frac{3\pi}{4}]$? (Don't forget to think about the end-points).

A. None
B. 1
C. 2
D. 3
E. 4
F. 5
G. 6
H. 7
I. 8
J. Infinitely Many



5. Let $f(x) = x^{\ln x}$. What is $f'(2)$

A. 1.10
B. 1.11
C. 1.12
D. 1.13
E. 1.14
F. 1.15
G. 1.16
H. 1.17
I. 1.18
J. 1.19

$$y = x^{\ln x}$$
$$\ln y = \ln x^{\ln x} = (\ln x)^2$$
$$y = e^{(\ln x)^2}$$
$$\frac{dy}{dx} = e^{(\ln x)^2} \cdot 2\ln x \cdot \frac{1}{x}$$
$$f'(2) = e^{(\ln 2)^2} \cdot \ln 2 = 1.1207$$

6. Evaluate $.33^3 + .33^4 + .33^5 + \ldots$

A. .01
B. .02
C. .03
D. .04
E. .05
F. .06
G. .07
H. .08
I. .09
J. .10

$$= .33^3 [1 + .33 + .33^2 + \ldots]$$
$$= \frac{.33^3}{1 - .33} = .0539$$

7. If $f(s) = s^3 + 2s - 7$, what is $(f^{-1})'(5)$?
(Note that $f(2) = 5$).

A. .010
B. .011
C. .012
D. .013
E. .014
F. .05
G. .06
H. .07
I. .08
J. .09

$$(f^{-1})'(5) = \frac{1}{f'(2)} = \frac{1}{3s^2+2}\Big|_{s=2}$$
$$= \frac{1}{14}$$
$$= .0714\ldots$$

8. Let

$$a_n = 4 + \frac{(-1)^n n}{n^3+1}, \quad b_n = 4 + \frac{(-1)^n n^2}{n^3+1}, \quad c_n = 4 + \frac{(-1)^n n^3}{n^3+1}.$$

Consider the assertions:
(I) $\lim_{n\to\infty} a_n$ exists. T
(II) $\lim_{n\to\infty} b_n$ exists. T
(III) $\lim_{n\to\infty} c_n$ exists. F

A. All 3 are true.
B. (I) and (II) are true, (III) is false.
C. (I) and (III) are true, (II) is false.
D. (II) and (III) are true, (I) is false.
E. (I) is true, (II) and (III) are false.
F. (II) is true, (I) and (III) are false.
G. (III) is true, (I) and (II) are false.
H. All three are false.

$$a_n = 4 + \frac{(-1)^n}{n^2+\frac{1}{n}} \to 4$$

$$b_n = 4 + \frac{(-1)^n}{n+\frac{1}{n^2}} \to 4$$

$$c_n = 4 + \frac{(-1)^n}{1+\frac{1}{n^3}}$$

For Even n, $c_n \to 5$
For odd n, $c_n \to 3$

9. Calculate

$$\lim_{x\to 0} \frac{\sin(0.3x)\sin(6x)}{x^2}.$$

A. 0
B. 0.6
C. 1.2
D. 1.8
E. 2.4
F. 3
G. 3.6
H. $-\infty$
I. ∞
J. Does not exist, and is not ∞ or $-\infty$.

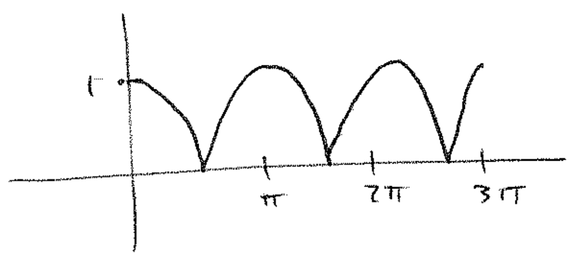
$$= \lim_{x\to 0} \frac{\sin(.3x)}{(.3)x} \frac{\sin(6x)}{6x}\, \underbrace{x(.3)x(6)}$$

$$\downarrow \qquad \downarrow \qquad =$$

$$1 \qquad 1 \qquad 1.8$$

10. Let $f : [0, 3\pi] \to [0, 2]$ be defined by $f(x) = (\cos(x))^2$. Which of the following is true?

A. f is bijective
B. f is one-to-one but not onto
C. f is onto but not one-to-one
D. f is neither one-to-one nor onto



11. What is

$$\sum_{j=1}^{900} j \ ? \qquad = \frac{900(901)}{2} = 405450$$

A. 405300
B. 405350
C. 405400
D. 405450
E. 405500
F. 405550
G. 405600
H. 405650
I. 405700
J. 405750

12. Let $f(x) = 2x + 11$ and $g(x) = x^3 + 1$, where f and g have domain and codomain $\mathbb{R}$. What is $f \circ (g^{-1})(9)$?

A. -10
B. -5
C. 0
D. 5
E. 10
F. 15
G. 20
H. 25
I. 30
J. 35

Let $g^{-1}(9) = x$

$x^3 + 1 = 9$

$x^3 = 8$

$x = 2$

$f(2) = 15$

13. Calculate

$$\frac{d}{dx}e^{\sin(x^2)}$$ $= e^{\sin(x^2)} \cdot \cos(x^2) \cdot 2x$

at $x = .5$.

At $x = .5$, get 1.2409

A. 1.20
B. 1.21
C. 1.22
D. 1.23
E. 1.24
F. 1.25
G. 1.26
H. 1.27
I. 1.28
J. 1.29

14. Calculate

$$\lim_{x\to\infty} e^{-2x}\ln(2x).$$ $= \lim_{x\to\infty} \frac{\ln 2x}{e^{2x}}$

$\overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{1}{2x}\cdot 2}{2e^{2x}}$ $\to 0$, $\to \infty$

$= 0$

A. 0
B. .25
C. .5
D. .75
E. 1.0
F. 1.25
G. 1.5
H. 1.75
I. 2.0
J. ∞

15. To solve the equation $x^3 + x + 1 = 0$ using the Newton-Raphson method, if the first estimate $x_1 = 0$, what is x_3?

A. -.71
B. -.72
C. -.73
D. -.74
E. -.75
F. -.76
G. -.77
H. -.78
I. -.79
J. -.80

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_2 = 0 - \frac{1}{1} = -1$$

$$x_3 = -1 - \frac{-1}{4} = -.75$$

16. Let $x(t)$ denote the position of a particle, which is initially at rest at time $t = 0$, and satisfies $x(0) = 0$. Suppose it experiences an acceleration of $3 - 2t$. What is $x(1)$?

A. 1.11
B. 1.12
C. 1.13
D. 1.14
E. 1.15
F. 1.16
G. 1.17
H. 1.18
I. 1.19
J. 1.20

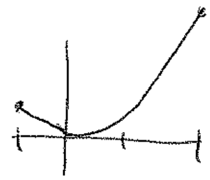
$$x''(t) = 3 - 2t$$

$$x'(t) = 3t - t^2$$

$$x(t) = \frac{3t^2}{2} - \frac{t^3}{3}$$

$$x(1) = \frac{3}{2} - \frac{1}{3} = 1.1667$$

17. Calculate the lower Riemann sum for the function $f(x) = 2x^2$ on the interval $[-0.5, 1.5]$ with the uniform partition of order 2, *i.e.* with $x_0 = -0.5, x_1 = 0.5, x_2 = 1.5$.



A. 0.1
B. 0.2
C. 0.3
D. 0.4
E. 0.5
F. 0.6
G. 0.7
H. 0.8
I. 0.9
J. 1.0

$$(1)\cdot\min_{[-.5,.5]} f \quad + (1)\min_{[.5,1.5]} f$$

$$= 1(2)(.5)^2 = .5$$

18. Calculate

$$\int_0^2 e^{3x}\,dx.$$

A. 130
B. 131
C. 132
D. 133
E. 134
F. 135
G. 136
H. 137
I. 138
J. 139

$$\frac{d}{dx} e^{3x} = 3e^{3x}$$

$$\therefore \frac{d}{dx} \frac{1}{3} e^{3x} = e^{3x}$$

$$\int_0^2 e^{3x}\,dx = \frac{1}{3} e^{3x} \Big|_0^2$$

$$= \frac{1}{3}(e^6 - 1)$$

$$= 134.143$$

19. Calculate

$$\int_1^3 x^2 - 3x + 5\,dx.$$

A. 6.1
B. 6.2
C. 6.3
D. 6.4
E. 6.5
F. 6.6
G. 6.7
H. 6.8
I. 6.9
J. 7.0

$$\frac{x^3}{3} - \frac{3x^2}{2} + 5x \Big|_1^3$$

$$= 9 - 13\tfrac{1}{2} + 15 - \left(\tfrac{1}{3} - \tfrac{3}{2} + 5\right)$$

$$= 6.\overline{6}$$

20. What is

$$\frac{d}{dx}\int_0^x e^{-2t^2}\,dt \quad = \quad e^{-2x^2}$$

when $x = \sqrt{2}$?

A. .01
B. .02
C. .03
D. .04
E. .05
F. .06
G. .07
H. .08
I. .09
J. .10

$$e^{-2(\sqrt{2})^2} = .0183$$