

Math 131, Exam 1, September 18th

This exam should have 16 multiple choice questions, and two hand-written questions. Each multiple-choice question is worth 5 points; each hand-written problem is worth 10 points.

Mark your ID number on the six blank lines on the top of your answer card, using one line for each digit. Print your name on the top of the card.

Write your name and student ID number on each of the hand-graded sheets.

Mark your answer card with a **PENCIL** by shading in the correct box.

You may not use a calculator. There is a table of functions on page 2.

You may not have any written aids.

Table of functions

x	$\sin x$	$\cos x$	$\tan x$	$\ln x$	e^x
0	0	1.0	0		1.0
0.1	0.100	0.995	0.100	-2.303	1.105
0.2	0.199	0.980	0.203	-1.609	1.221
0.3	0.296	0.955	0.309	-1.204	1.350
0.4	0.389	0.921	0.423	-0.916	1.492
0.5	0.479	0.878	0.546	-0.693	1.649
0.6	0.565	0.825	0.684	-0.511	1.822
0.7	0.644	0.765	0.842	-0.357	2.014
0.8	0.717	0.697	1.030	-0.223	2.226
0.9	0.783	0.622	1.260	-0.105	2.460
1	0.841	0.540	1.557	0.000	2.718
1.1	0.891	0.454	1.965	0.095	3.004
1.2	0.932	0.362	2.572	0.182	3.320
1.3	0.964	0.267	3.602	0.262	3.669
1.4	0.985	0.170	5.798	0.336	4.055
1.5	0.997	0.071	14.101	0.405	4.482
1.6	1.000	-0.029	-34.233	0.470	4.953
1.7	0.992	-0.129	-7.697	0.531	5.474
1.8	0.974	-0.227	-4.286	0.588	6.050
1.9	0.946	-0.323	-2.927	0.642	6.686
2	0.909	-0.416	-2.185	0.693	7.389
2.1	0.863	-0.505	-1.710	0.742	8.166
2.2	0.808	-0.589	-1.374	0.788	9.025
2.3	0.746	-0.666	-1.119	0.833	9.974
2.4	0.675	-0.737	-0.916	0.875	11.023
2.5	0.598	-0.801	-0.747	0.916	12.182
2.6	0.516	-0.857	-0.602	0.956	13.464
2.7	0.427	-0.904	-0.473	0.993	14.880
2.8	0.335	-0.942	-0.356	1.030	16.445
2.9	0.239	-0.971	-0.246	1.065	18.174
3	0.141	-0.990	-0.143	1.099	20.086
3.1	0.042	-0.999	-0.042	1.131	22.198

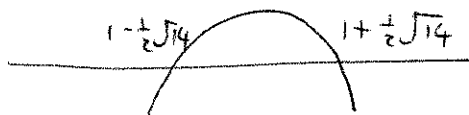
1. The largest domain of the real-valued function $\sqrt{5 + 4x - 2x^2}$ is an interval $[a, b]$. The length $b - a$ is:

- A. $\sqrt{9}$
- B. $\sqrt{10}$
- C. $\sqrt{11}$
- D. $\sqrt{12}$
- E. $\sqrt{13}$
- ☒ F. $\sqrt{14}$
- G. $\sqrt{15}$
- H. $\sqrt{16}$
- I. $\sqrt{17}$
- J. $\sqrt{18}$

Need: $5 + 4x - 2x^2 \geq 0$.

Roots are $\frac{-4 \pm \sqrt{16 + 40}}{-4}$

$= 1 \pm \frac{1}{2}\sqrt{14}$



2. Define a sequence inductively by $f_1 = 2$ and $f_{n+1} = 1 - f_n^2$. What is f_4 ?

- A. -65
- B. -64
- ☒ C. -63
- D. -62
- E. -61
- F. 61
- G. 62
- H. 63
- I. 64
- J. 65

$f_2 = 1 - (2)^2 = -3$

$f_3 = 1 - (-3)^2 = -8$

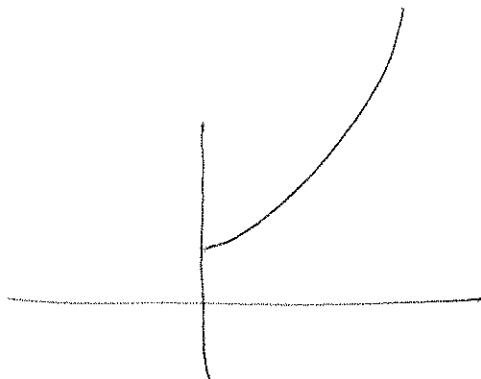
$f_4 = 1 - (-8)^2 = -63$

3. Consider the function

$$\begin{aligned} f : [0, \infty) &\rightarrow [1, \infty) \\ x &\mapsto x^2 + 1. \end{aligned}$$

Which of the following is true:

- ☒ A. f is bijective.
- ☐ B. f is one-to-one but not onto.
- ☐ C. f is onto but not one-to-one.
- ☐ D. f is neither one-to-one nor onto.

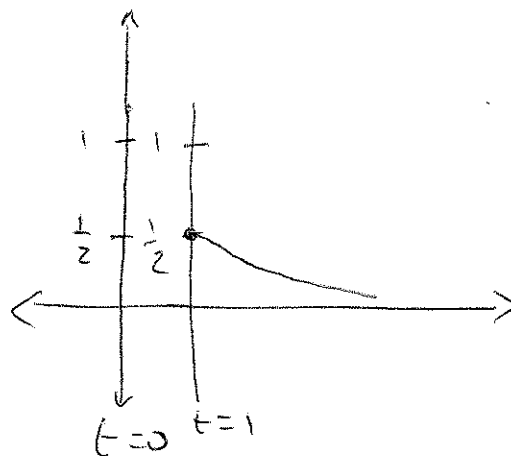


4. Consider the function

$$\begin{aligned} f : [1, \infty) &\rightarrow (0, 1) \\ t &\mapsto \frac{1}{1 + t^2}. \end{aligned}$$

Which of the following is true:

- ☐ A. f is bijective.
- ☒ B. f is one-to-one but not onto.
- ☐ C. f is onto but not one-to-one.
- ☐ D. f is neither one-to-one nor onto.



5. Suppose $0 < \theta < \frac{\pi}{2}$, and $\cos(\theta) = 0.921$. (See table of functions) Then $\sin(\theta)$ is, to one significant digit:

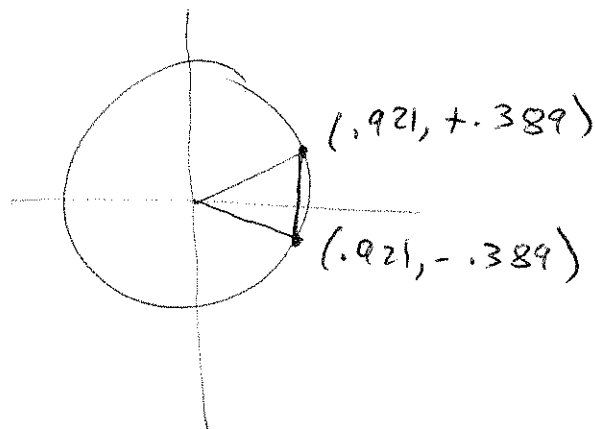
- A. -0.4
- B. -0.3
- C. -0.2
- D. -0.1
- E. 0
- F. 0.1
- G. 0.2
- H. 0.3
- I. 0.4
- J. 0.5

$$\theta = 0.4 \text{ radians}$$

$$\sin \theta = .389$$

6. Suppose $\frac{3\pi}{2} < \theta < 2\pi$, and $\cos(\theta) = 0.921$. Then $\sin(\theta)$ is, to one significant digit:

- A. -0.4
- B. -0.3
- C. -0.2
- D. -0.1
- E. 0
- F. 0.1
- G. 0.2
- H. 0.3
- I. 0.4
- J. 0.5



$$-.389$$

7. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \frac{1}{2}x^3 - 2$. What is $f^{-1}(2)$?

A. -4

B. -3

C. -2

D. -1

E. 0

F. 1

G. 2

H. 3

I. 4

J. 5

$$f(x) = 2$$

$$\frac{1}{2}x^3 - 2 = 2$$

$$\frac{1}{2}x^3 = 4$$

$$x^3 = 8$$

$$x = 2$$

8. Let $f(x) = 3x^2 + 1$ and $g(x) = x^2 + 3$. Then $f \circ g(1) - g \circ f(1)$ is:

A. -40

B. -30

C. -20

D. -10

E. 0

F. 10

G. 20

H. 30

I. 40

J. 50

$$g(1) = 4$$

$$f(g(1)) = f(4) = 49$$

$$f(1) = 4$$

$$g(f(1)) = g(4) = 19$$

$$49 - 19 = 30$$

9. Consider the following three functions:

$$\begin{aligned} f(x) &= 2x^5 - 3x^2 + 2 && \text{Neither} \\ g(x) &= e^{-x^2} && \text{Even} \\ h(x) &= 1 + \sin x. && \text{Neither} \end{aligned}$$

Which of the following statements is true:

- A. All 3 functions are even.
- B. Two functions are even, one is neither even nor odd.
- C. Two functions are even, one is odd.
- D. One function is even, two functions are odd.
- E. One function is even, one is odd, one is neither even nor odd.
- ☒ F. One function is even, two functions are neither even nor odd.
- G. All 3 functions are odd.
- H. Two functions are odd, one is neither even nor odd.
- I. One function is odd, two are neither even nor odd.
- J. None of the functions is either even or odd.

10. The function $f(x) = 1.8 + 5.21 \sin(\frac{\pi}{5}x - \frac{3\pi}{10})$ is largest at which of the following numbers:

- A. 0
- B. 1
- C. 2
- D. 3
- ☒ E. 4
- F. 5
- G. 6
- H. 7
- I. 8
- J. 9

Sin is largest at $\frac{\pi}{2} + 2n\pi, n \in \mathbb{Z}$

$\sin(\frac{\pi}{5}x - \frac{3\pi}{10})$ is largest

$$y: \frac{\pi}{5}x - \frac{3\pi}{10} = \frac{\pi}{2} + 2n\pi$$

$$x \frac{10}{\pi} : 2x - 3 = 5 + 20n$$

$$2x = 8 + 20n$$

$$x = 4 + 10n$$

11. Calculate the slope of the line segment joining the points (1, 3) and (3, -3).

A. -4

B. -3

C. -2

D. -1

E. 0

F. 1

G. 2

H. 3

I. 4

J. 5

$$\frac{-3-3}{3-1} = \frac{-6}{2} = -3$$

12. Solve the equation $e^x = 1.5$. Which of these answers is closest?

A. .30

B. .32

C. .34

D. .36

E. .38

F. .40

G. .42

H. .44

I. .46

J. .48

$$e^x = 1.5$$

$$x = \ln(1.5) = .405$$

13. Using the Table of functions, calculate the cube root of 2.46. (Hint: This should require very little arithmetic). Which of these answers is closest?

- A. 1.05
- B. 1.1
- C. 1.15
- D. 1.2
- E. 1.25
- F. 1.3
- G. 1.35
- H. 1.4
- I. 1.45
- J. 1.5

$$y = (2.46)^{1/3}$$

$$= e^{\frac{1}{3} \ln(2.46)}$$

From p. 2, $2.46 = e^{0.9}$

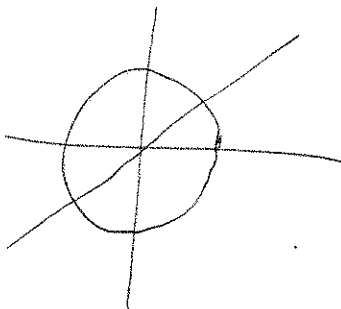
So $(2.46)^{1/3} = (e^{0.9})^{1/3} = e^{0.3}$

(Equivalently, $\ln(2.46) = 0.9$) = 1.350

14. If $0 \leq \theta \leq \pi$ and $\tan \theta = 1$, then θ is:

- A. 0
- B. $\pi/8$
- C. $\pi/6$
- D. $\pi/4$
- E. $3\pi/8$
- F. $\pi/3$
- G. $\pi/2$
- H. $5\pi/8$
- I. $2\pi/3$
- J. $3\pi/4$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = 1 \quad \text{if } \theta = \frac{\pi}{4} + n\pi$$



15. Consider the three statements:

- (I) Every real number has a real cube root.
- (II) Every real number has a real fourth root.
- (III) Every real number has a real logarithm.

T
F
F

Then:

- A. All 3 are true.
- B. (I) and (II) are true, (III) is false.
- C. (I) and (III) are true, (II) is false.
- D. (II) and (III) are true, (I) is false.
- ☒ E. (I) is true, (II) and (III) are false.
- F. (II) is true, (I) and (III) are false.
- G. (III) is true, (I) and (II) are false.
- H. All three are false.

16. Consider the three statements:

- (I) For every real number x , $\cos x$ is a real number.
- (II) For every positive real number x , $\sin x$ is a real number.
- (III) For every real number x , $\tan x$ is a real number.

T
T
F

Then:

- A. All 3 are true.
- ☒ B. (I) and (II) are true, (III) is false.
- C. (I) and (III) are true, (II) is false.
- D. (II) and (III) are true, (I) is false.
- E. (I) is true, (II) and (III) are false.
- F. (II) is true, (I) and (III) are false.
- G. (III) is true, (I) and (II) are false.
- H. All three are false.

Math 131 Hand-graded Problems

Student Name:

Student ID:

Problem A: Find the inverse function of

$$f(x) = \frac{x+1}{2x+1}.$$

$$y = \frac{x+1}{2x+1}$$

$$2xy + y = x + 1$$

$$2xy - x = 1 - y$$

$$x(2y-1) = 1-y$$

$$x = \frac{1-y}{2y-1}$$

$$f^{-1}(y) = \frac{1-y}{2y-1}$$

Problem B: Solve the inequality

$$|x| + |x+2| < 12.$$

There are 3 cases: $x \leq -2$, $-2 \leq x \leq 0$, $x \geq 0$

Case (I): If $x \leq -2$, inequality is $-x - (x+2) < 12$

$$\begin{aligned} -2x &< 14 \\ x &> -7 \end{aligned}$$

Case (II): $-2 \leq x \leq 0$:

$$-x + x + 2 < 12$$

$2 < 12$: Always true

Case (III): $x \geq 0$: $x + x + 2 < 12 \Rightarrow 2x < 10 \Rightarrow x < 5.$

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Conclude: Inequality holds if $-7 < x < 5.$