Math 131, Exam 1, September 19$^{\text{th}}$

This exam should have 16 multiple choice questions, and two hand-written questions. Each multiple-choice question is worth 5 points; each hand-written problem is worth 10 points.

Mark your ID number on the six blank lines on the top of your answer card, using one line for each digit. Print your name on the top of the card.

Write your name and student ID number on each of the hand-graded sheets.

Choose the answer that is *closest* to the solution — the exact solution may not be on the list. Mark your answer card with a **PENCIL** by shading in the correct box.

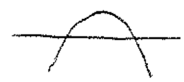
You may use a calculator, but not one that has a graphing function.

You may not have any written aids.

1. The largest domain of the real-valued function $\sqrt{5+4x-2x^2}$ is an interval $[a,b]$. To two decimal places, the value of a is:

A. -0.80
B. -0.81
C. -0.82
D. -0.83
E. -0.84
F. -0.85
G. -0.86
H. -0.87
I. -0.88
J. -0.89

Need $q(x) = 5+4x-2x^2 \geq 0$.

q looks like , so $[a,b]$ is interval between the roots, which are

$$\frac{-4 \pm \sqrt{16+40}}{-40}.$$

So $a = -.8708$

2. Define a sequence inductively by $f_1 = 0.9$ and $f_{n+1} = 1 + f_n^2$. What is f_4?

A. 19.21
B. 19.22
C. 19.23
D. 19.24
E. 19.25
F. 19.26
G. 19.27
H. 19.28
I. 19.29
J. 19.30

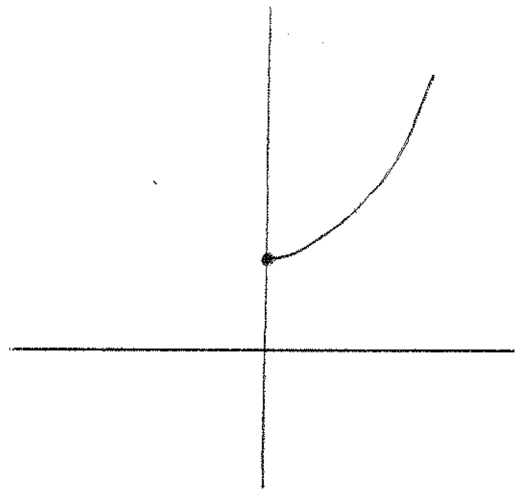
$f_2 = 1 + (0.9)^2 = 1.81$

$f_3 = 1 + (1.81)^2 = 4.2761$

$f_4 = 1 + (4.2761)^2 = 19.28503$

3. Consider the function

$$\begin{aligned} f : [0,\infty) &\to [1,\infty) \\ x &\mapsto x^3+1. \end{aligned}$$



Which of the following is true:

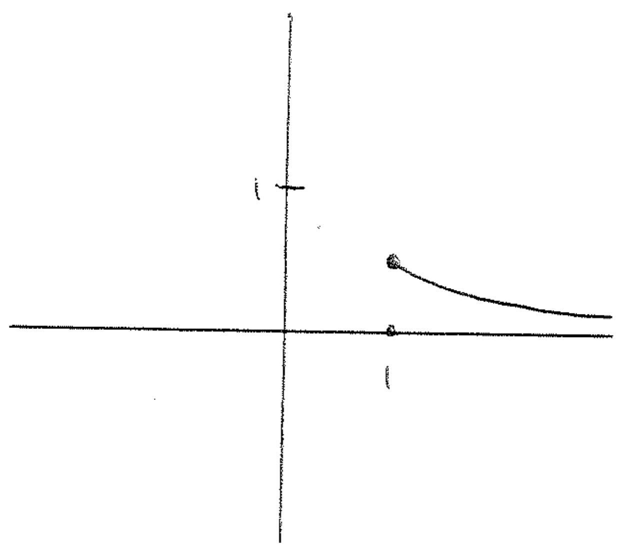
A. f is bijective.
B. f is one-to-one but not onto.
C. f is onto but not one-to-one.
D. f is neither one-to-one nor onto.

4. Consider the function

$$\begin{aligned} f : [1,\infty) &\to [0,1) \\ t &\mapsto \frac{1}{1+t^2}. \end{aligned}$$

Which of the following is true:

A. f is bijective.
B. f is one-to-one but not onto.
C. f is onto but not one-to-one.
D. f is neither one-to-one nor onto.



5. Suppose $0 < \theta < \frac{\pi}{2}$, and $\sin(\theta) = 1/5$. Then $\cos(\theta)$ is

A. 0.971
B. 0.972
C. 0.973
D. 0.974
E. 0.975
F. 0.976
G. 0.977
H. 0.978
I. 0.979
J. 0.980

$\cos^2\theta = 1 - \sin^2\theta = .96$

$\cos\theta > 0$

$\therefore \cos\theta = +\sqrt{.96} = .9797959$

6. One ounce is 28.35 grams. To four significant digits, how precisely must weight be measured in ounces to guarantee accuracy of 0.01 gram?

A. 0.00003527
B. 0.0003527
C. 0.003527
D. 0.03527
E. 0.3527
F. 0.00002835
G. 0.0002835
H. 0.002835
I. 0.02835
J. 0.2835

Let W be weight in grams,
M " " " ounces.

$W = 28.35\,M$

$M = \frac{1}{28.35} W$

So 0.01 grams corresponds to $(0.01) \times \frac{1}{28.35}$ ounces

7. Let

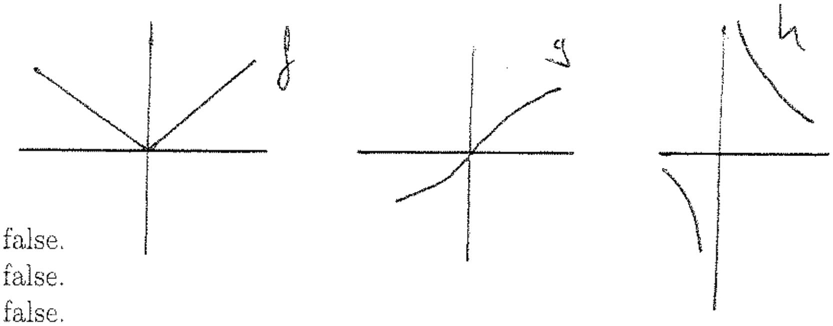
$$f(x) = |x|, \; g(x) = \frac{x^{5/3}}{|x|}, \; h(x) = \frac{|x|}{x^{5/3}}.$$

Consider the assertions:

T (I) $\lim_{x\to 0} f(x)$ exists.

T (II) $\lim_{x\to 0} g(x)$ exists.

F (III) $\lim_{x\to 0} h(x)$ exists.



A. All 3 are true.
B. (I) and (II) are true, (III) is false.
C. (I) and (III) are true, (II) is false.
D. (II) and (III) are true, (I) is false.
E. (I) is true, (II) and (III) are false.
F. (II) is true, (I) and (III) are false.
G. (III) is true, (I) and (II) are false.
H. All three are false.

8. Let

$$f(x) = \frac{x^2-3x+2}{x-3}, \; g(x) = \frac{x^2-2x+2}{x-2}, \; h(x) = \frac{x^2-3x+2}{x-2}.$$

Consider the assertions:

T (I) $\lim_{x\to 2} f(x)$ exists.

F (II) $\lim_{x\to 2} g(x)$ exists.

T (III) $\lim_{x\to 2} h(x)$ exists.

$x^2-3x+2 = (x-1)(x-2)$

$\therefore$ if $x \neq 2$, $h(x) = x-1$

A. All 3 are true.
B. (I) and (II) are true, (III) is false.
C. (I) and (III) are true, (II) is false.
D. (II) and (III) are true, (I) is false.
E. (I) is true, (II) and (III) are false.
F. (II) is true, (I) and (III) are false.
G. (III) is true, (I) and (II) are false.
H. All three are false.

9. Let

$$f(x) = \frac{\sin x}{2x}, \quad g(x) = \frac{\sin 2x}{x}, \quad h(x) = \frac{\sin(x^2)}{x}.$$

Consider the assertions:

(I) $\lim_{x\to 0} f(x)$ exists.
(II) $\lim_{x\to 0} g(x)$ exists.
(III) $\lim_{x\to 0} h(x)$ exists.

$\lim_{x\to 0} f(x) = \frac{1}{2}$

$\lim_{x\to 0} g(x) = \lim_{x\to 0} \frac{2 \sin 2x}{2x} = 2$

$\lim_{x\to 0} h(x) = \lim_{x\to 0} \frac{\sin(x^2)}{x^2} \cdot x$

$= 1 \cdot 0$

$= 0$

A. All 3 are true.
B. (I) and (II) are true, (III) is false.
C. (I) and (III) are true, (II) is false.
D. (II) and (III) are true, (I) is false.
E. (I) is true, (II) and (III) are false.
F. (II) is true, (I) and (III) are false.
G. (III) is true, (I) and (II) are false.
H. All three are false.

10. Consider the function $f(x) = \sin x$ on the interval $I = [-\frac{\pi}{3}, \frac{\pi}{4}]$. A consequence of the Intermediate Value Theorem is that there is a largest interval $[\alpha, \beta]$ with the property that for every γ in $[\alpha, \beta]$, there exists some c in I such that $f(c) = \gamma$. Which of these numbers is closest to α?

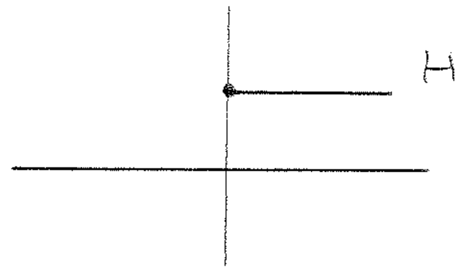
A. -1
B. -0.8
C. -0.6
D. -0.4
E. -0.2
F. 0
G. 0.2
H. 0.4
I. 0.6
J. 0.8

$\alpha = \sin(-\pi/3) = -\frac{\sqrt{3}}{2} = -.866$

$\beta = \sin(\pi/4)$

11. Let $H : \mathbb{R} \to \mathbb{R}$ be defined by

$$H(x) = \begin{cases} 0 & x < 0; \\ 1 & x \geq 0. \end{cases}$$



Consider the assertions:

(I) $\lim_{x \to 0^+} H(x)$ exists. $= 1$

(II) $\lim_{x \to 0^-} H(x)$ exists. $= 0$

(III) $\lim_{x \to 0} H(x)$ exists. No.

A. All 3 are true.
B. (I) and (II) are true, (III) is false.
C. (I) and (III) are true, (II) is false.
D. (II) and (III) are true, (I) is false.
E. (I) is true, (II) and (III) are false.
F. (II) is true, (I) and (III) are false.
G. (III) is true, (I) and (II) are false.
H. All three are false.

12. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^3 + 1$. What is $f^{-1}(5)$?

A. 1.50
B. 1.51
C. 1.52
D. 1.53
E. 1.54
F. 1.55
G. 1.56
H. 1.57
I. 1.58
J. 1.59

$x^3 + 1 = 5$

$x^3 = 4$

$x = 1.5874$

13. Let $f(x) = 2x^2 + 1$ and $g(x) = 2.1x^2 + 0.9$. Then $f \circ g(0.9)$ is:

A. 14.5
B. 14.6
C. 14.7
D. 14.8
E. 14.9
F. 15.0
G. 15.1
H. 15.2
I. 15.3
J. 15.4

$g(0.9) = 2.601$

$f(2.601) = 14.53$

14. The function $f(x) = 3.4 + 5.21\sin(\frac{\pi}{5}x - \frac{3\pi}{10})$ is largest at which of the following numbers:

A. 0
B. 1
C. 2
D. 3
E. 4
F. 5
G. 6
H. 7
I. 8
J. 9

$\sin(x)$ is largest when $x = \frac{\pi}{2} + 2\pi n$, $n \in \mathbb{Z}$

$\sin(\frac{\pi}{5}x)$ " " " $x = \frac{5}{2} + 10n$

$\sin(\frac{\pi}{5}(x - \frac{3}{2}))$ " " " $x - \frac{3}{2} = \frac{5}{2} + 10n$

$x = 4 + 10n$

15.Calculate the slope of the line segment joining the points $(1, 2.8)$ and $(1.07, 2.83)$. Which of these numbers is closest?

A. -1
B. -0.8
C. -0.6
D. -0.4
E. -0.2
F. 0
G. 0.2
H. 0.4
I. 0.6
J. 0.8

$$\frac{2.83-2.8}{1.07-1.0} = \frac{.03}{.07} = .4286$$

16. Let $f : \mathbb{R} \to \mathbb{R}$. Consider the assertions:
(I) If f is continuous, then $|f|$ is continuous. T
(II) If f^2 is continuous, then f is continuous. F
(III) If f is continuous, then $\dfrac{f}{1+f^2}$ is continuous. T

A. All 3 are true.
B. (I) and (II) are true, (III) is false.
C. (I) and (III) are true, (II) is false.
D. (II) and (III) are true, (I) is false.
E. (I) is true, (II) and (III) are false.
F. (II) is true, (II) and (III) are false.
G. (III) is true, (I) and (II) are false.
H. All three are false.

Ad II: Consider $f(x) = \begin{cases} +1 & x \geq 0 \\ -1 & x < 0. \end{cases}$

Math 131 Hand-graded Problems

Student Name:

Student ID:

Problem A: Give a formal definition of the statement

$$\lim_{x \to c} f(x) = L.$$

$\forall \varepsilon > 0, \exists \delta > 0$ such that

$0 < |x - c| < \delta \Rightarrow |f(x) - L| < \varepsilon$.

...words: For every $\varepsilon > 0$, there exists $\delta > 0$ such that if $0 < |x-c| < \delta$ then $|f(x) - L| < \varepsilon$

Problem B: Prove that

$$p(x) = 3x^5 - 7x^2 + 1$$

has at least 3 real roots, and that each of the intervals $(-1, 0)$, $(0, 1)$ and $(1, 2)$ contains a root of p.

p is continuous, since it is a polynomial

$p(-1) = -9$

$p(0) = 1$

$p(1) = -3$

$p(2) = 69$

p must have a root between any two points where its sign is opposite

So p must have roots between -1 and 0, between 0 & 1, and between 1 & 2.