Math 131, Exam 1, October 16^{th}

This exam should have 16 multiple choice questions, and two hand-written questions. Each multiple-choice question is worth 5 points; each hand-written problem is worth 10 points.

Mark your ID number on the six blank lines on the top of your answer card, using one line for each digit. Print your name on the top of the card.

Write your name and student ID number on each of the hand-graded sheets.

Mark your answer card with a **PENCIL** by shading in the correct box.

You may not use a calculator. There is a table of functions on page 2.

You may not have any written aids.

Table of functions

x	$\sin x$	$\cos x$	$\tan x$	$\ln x$	e^x
0	0	1.0	0		1.0
0.1	0.100	0.995	0.100	-2.303	1.105
0.2	0.199	0.980	0.203	-1.609	1.221
0.3	0.296	0.955	0.309	-1.204	1.350
0.4	0.389	0.921	0.423	-0.916	1.492
0.5	0.479	0.878	0.546	-0.693	1.649
0.6	0.565	0.825	0.684	-0.511	1.822
0.7	0.644	0.765	0.842	-0.357	2.014
0.8	0.717	0.697	1.030	-0.223	2.226
0.9	0.783	0.622	1.260	-0.105	2.460
1	0.841	0.540	1.557	0.000	2.718
1.1	0.891	0.454	1.965	0.095	3.004
1.2	0.932	0.362	2.572	0.182	3.320
1.3	0.964	0.267	3.602	0.262	3.669
1.4	0.985	0.170	5.798	0.336	4.055
1.5	0.997	0.071	14.101	0.405	4.482
1.6	1.000	-0.029	-34.233	0.470	4.953
1.7	0.992	-0.129	-7.697	0.531	5.474
1.8	0.974	-0.227	-4.286	0.588	6.050
1.9	0.946	-0.323	-2.927	0.642	6.686
2	0.909	-0.416	-2.185	0.693	7.389
2.1	0.863	-0.505	-1.710	0.742	8.166
2.2	0.808	-0.589	-1.374	0.788	9.025
2.3	0.746	-0.666	-1.119	0.833	9.974
2.4	0.675	-0.737	-0.916	0.875	11.023
2.5	0.598	-0.801	-0.747	0.916	12.182
2.6	0.516	-0.857	-0.602	0.956	13.464
2.7	0.427	-0.904	-0.473	0.993	14.880
2.8	0.335	-0.942	-0.356	1.030	16.445
2.9	0.239	-0.971	-0.246	1.065	18.174
3	0.141	-0.990	-0.143	1.099	20.086
3.1	0.042	-0.999	-0.042	1.131	22.198

1. Calculate: $\lim_{x\to\infty} \frac{6x^2-8x+1}{2x^2-3x+5}$

A. 0
B. 1/5
C. 1/3
D. 1/2
E. 1
F. 6/5
G. 8/5
H. 8/3
I. 3
J. does not exist

$$= \lim_{x\to\infty} \frac{6 - 8/x + 1/x^2}{2 - 3/x + 5/x^2} = 3$$

2. Evaluate

$$\lim_{t\to 2} \frac{t^2-4}{t^3-8}.$$

A. $-\infty$
B. -1
C. -1/2
D. -1/3
E. -1/4
F. 0
G. 1/4
H. 1/3
I. 1/2
J. ∞

$$= \lim_{t\to 2} \frac{(t-2)(t+2)}{(t-2)(t^2+2t+4)}$$

$$= \lim_{t\to 2} \frac{t+2}{t^2+2t+4} = \frac{4}{12}$$

3. Consider the three statements:

(I) There exist functions that have three horizontal asymptotes.

(II) If f is a continuous function defined on the real line, then f is continuous from the right at each point.

(III) If the function g is continuous at a, then so is g^2.

Then:

A. All 3 are true.
B. (I) and (II) are true, (III) is false.
C. (I) and (III) are true, (II) is false.
D. (II) and (III) are true, (I) is false.
E. (I) is true, (II) and (III) are false.
F. (II) is true, (I) and (III) are false.
G. (III) is true, (I) and (II) are false.
H. All three are false.

I: Impossible, since can have at most one limit at $+\infty$ & another at $-\infty$

II: True

III: True (from limit laws)

4. The position of a particle is given by the function $p(t) = 3\cos(\pi t) + 7t$ for $t \in \mathbb{R}$. Calculate the average velocity of the particle over the time period $1 \leq t \leq 4$.

A. 8
B. 8.5
C. 9
D. 9.5
E. 10
F. 10.5
G. 11
H. 11.5
I. $+\infty$
J. $-\infty$

$p(4) = 3\cos(4\pi) + 28 = 31$

$p(1) = 3\cos\pi + 7 = 4$

$\frac{p(4) - p(1)}{4-1} = \frac{27}{3} = 9$

5. Let $y = mx + b$ be the tangent line to the curve $f(x) = 4x^2 + 2$ at the point $(0, 2)$. What is $m + b$?

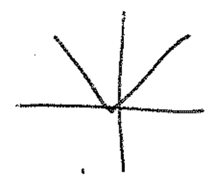
A. 0
B. 1
C. 2
D. 3
E. 4
F. 5
G. 6
H. 7
I. 8
J. undefined

$$f'(0) = \lim_{h \to 0} \frac{4h^2 + 2 - 2}{h} = 0$$

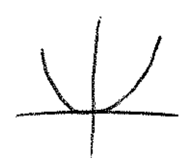
$\therefore$ $y = 2$ is tangent

6. Let $f, g : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = |x|$ and $g(x) = (f(x))^2$. Consider the following three statements.
(I) f is differentiable on all of $\mathbb{R}$.
(II) f is continuous on all of $\mathbb{R}$.
(III) g is differentiable on all of $\mathbb{R}$.
Then:

A. All 3 are true.
B. (I) and (II) are true, (III) is false.
C. (I) and (III) are true, (II) is false.
D. (II) and (III) are true, (I) is false.
E. (I) is true, (II) and (III) are false.
F. (II) is true, (I) and (III) are false.
G. (III) is true, (I) and (II) are false.
H. All three are false.

f:


Continuous, not differentiable

$g(x) = x^2$


is smooth

7. What's the value of

$$\lim_{x\to 0^+} x \tan^{-1}\frac{1}{x} \quad ?$$

A. $-\infty$
B. $-\frac{\pi}{2}$
C. $-\sqrt{2}$
D. -1
E. 0
F. 1
G. $\sqrt{2}$
H. $\frac{\pi}{2}$
I. ∞
J. π

$$= \lim_{x\to 0^+} x \cdot \lim_{x\to 0^+} \tan^{-1}\frac{1}{x}$$

$$= 0 \cdot \frac{\pi}{2}$$

$$= 0$$

8. Find the largest value of δ such that $|x-1| < \delta$ guarantees that $|\frac{5x-5}{10}| < 0.1$.

A. 0.02
B. 0.03
C. 0.04
D. 0.05
E. 0.2
F. 0.3
G. 0.4
H. 0.5
I. 0.1
J. 1

$$|x-1| < \delta \iff \underbrace{\tfrac{1}{2}|x-1|}_{=\left|\frac{5x-5}{10}\right|} < \delta/2$$

$$\therefore \frac{\delta}{2} = 0.1$$

$$\delta = 0.2$$

9. By looking at the table of functions at the beginning of the exam, the equation

$$\cos(x) = \tan(x)$$

has a solution x in which interval?

A. [0,0.1]
B. [0.1,0.2]
C. [0.2,0.3]
D. [0.3,0.4]
E. [0.4,0.5]
F. [0.5,0.6]
G. [0.6,0.7]
H. [0.7,0.8]
I. [0.8,0.9]
J. [0.9,1.0]

Cos x - tan x is continuous, positive at .6, negative at .7
So by the intermediate value theorem it has a root in (.6, .7)

10. Mike throws a ball in the air. The height of the ball t seconds later is given by $y = 14t - t^2$. What is the value of t when the velocity of the ball is 0?

A. 2
B. 4
C. 5
D. ~~6~~
E. 7
F. 8
G. 9
H. 11

$$y'(t) = \lim_{h \to 0} \frac{14(t+h) - (t+h)^2 - (14t - t^2)}{h}$$

$$= \lim_{h \to 0} \frac{14h - 2th - h^2}{h} = 14 - 2t$$

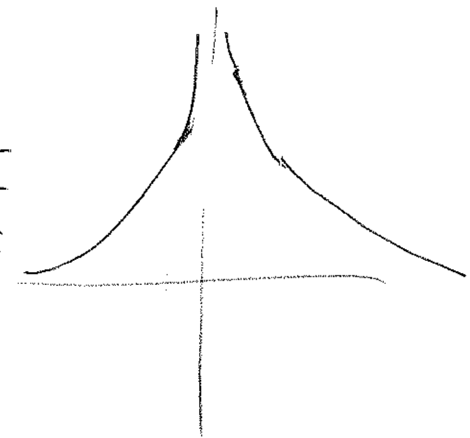
$y'(t) = 0$ when $14 - 2t = 0$

$14 = 2t$

$7 = t$

11. Consider the function $f(x) = \frac{1}{x^2}$.

Which of the following is true:
I. The graph of f has a vertical asymptote. T
II. The graph of f has a horizontal asymptote. T
III. The graph of f intersects the x-axis. F



A. All 3 are true.
B. (I) and (II) are true, (III) is false.
C. (I) and (III) are true, (II) is false.
D. (II) and (III) are true, (I) is false.
E. (I) is true, (II) and (III) are false.
F. (II) is true, (I) and (III) are false.
G. (III) is true, (I) and (II) are false.
H. All three are false.

12. What is $\lim_{x \to 3^-} \frac{2x}{x-3}$
A. $-\infty$
B. -6
C. -3
D. -2
E. $-2/3$
F. 0
G. 2/3
H. 2
I. 3
J. ∞

If x is close to 3 and less than 3,
$x-3$ is small (in magnitude) and negative
$\frac{1}{x-3}$ is large in magnitude and negative

$\therefore \frac{2x}{x-3} \to -\infty$ as $x \to 3^-$

13. Let $C(x)$ be the cost to produce x yards of cloth. Suppose cloth can be sold for \$2 per yard. Suppose that current production is scheduled to be 1000 yards, and that $C(1000) = \$1500$, and $C'(1000) = \$2.50/\text{yard}$. Consider the following 3 statements:

(I) The average production cost is \$1.50/yard.
(II) The average production cost is \$2.50/yard.
(III) It would be profitable to increase production a little.
Then:

A. All 3 are true.
B. (I) and (II) are true, (III) is false.
C. (I) and (III) are true, (II) is false.
D. (II) and (III) are true, (I) is false.
E. (I) is true, (II) and (III) are false.
F. (II) is true, (I) and (III) are false.
G. (III) is true, (I) and (II) are false.
H. All three are false.

$C'(1000) > \$2$
$\therefore$ III False

14. Suppose

$$\lim_{x\to 2} f(x) = 3, \lim_{x\to 3} f(x) = 2, \lim_{x\to 4} f(x) = 4$$
$$\lim_{x\to 2} g(x) = 4, \lim_{x\to 3} g(x) = 3, \lim_{x\to 4} g(x) = 2.$$

Then

$$\lim_{x\to 2} f \circ g(x)$$

is :

A. 0
B. 1
C. 2
D. 3
E. 4
F. 6
G. 8
H. 9
I. 12
J. 24

$4 = \lim_{x\to 2} g(x)$

Let $u = g(x)$

$\lim_{x\to 2} f \circ g(x) = \lim_{u\to 4} f(u) = 4$

15. Let $H : \mathbb{R} \to \mathbb{R}$ be defined by

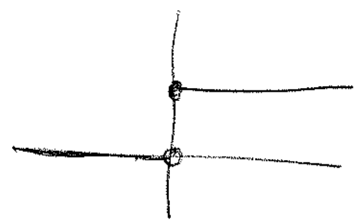
$$H(x) = \begin{cases} 0 & x < 0; \\ 1 & x \geq 0. \end{cases}$$

Consider the assertions:

(I) $\lim_{x\to 0^+} H(x)$ exists. T

(II) $\lim_{x\to 0^-} H(x)$ exists. T

(III) $\lim_{x\to 0} H(x)$ exists. F



A. All 3 are true.
B. (I) and (II) are true, (III) is false.
C. (I) and (III) are true, (II) is false.
D. (II) and (III) are true, (I) is false.
E. (I) is true, (II) and (III) are false.
F. (II) is true, (I) and (III) are false.
G. (III) is true, (I) and (II) are false.
H. All three are false.

16. Let h be the number of horizontal asymptotes, and v the number of vertical asymptotes, of the graph of

$$y = \frac{x^3 - 4x^2 + 8}{(x-4)(x^2+4)}.$$

(If the function has the same limit at $-\infty$ and $+\infty$, we will count that as just one horizontal asymptote). Then $v + 3h$ is:

A. 0
B. 1
C. 2
D. 3
E. 4
F. 5
G. 6
H. 7
I. 8
J. 9

$$\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{1 - 4/x + 8/x^3}{(1-\frac{4}{x})(1+4/x^2)} = 1$$

$$= \lim_{x\to-\infty} y$$

$\therefore h = 1$

Vertical asymptote at 4

$\therefore v = 1$

$\therefore v + 3h = 4$

Math 131 Hand-graded Problems

Student Name:

Student ID:

Problem A: Give a rigorous definition of what it means to say

$$\lim_{x\to a} f(x) = L.$$

For every $\varepsilon > 0$, there exists $\delta > 0$ such that if $0 < |x-a| < \delta$ then $|f(x) - L| < \varepsilon$.

Problem B: Evaluate

$$\lim_{x\to 0} \frac{|3x-1| - |3x+1|}{x}. \qquad *$$

For x close to 0, $3x-1 < 0$ & $3x+1 > 0$.

$\therefore |3x-1| = -(3x-1)$ and $|3x+1| = 3x+1$

So $* = \lim\limits_{x \to 0} \dfrac{-(3x-1)-(3x+1)}{x} = \lim\limits_{x\to 0} \dfrac{-6x}{x} = -6$