Math 131, Exam 2, October $17^{th}$

This exam should have 16 multiple choice questions, and two hand-written questions. Each multiple-choice question is worth 5 points; each hand-written problem is worth 10 points.

Mark your ID number on the six blank lines on the top of your answer card, using one line for each digit. Print your name on the top of the card.

Write your name and student ID number on each of the hand-graded sheets.

Choose the answer that is *closest* to the solution — the exact solution may not be on the list. Mark your answer card with a **PENCIL** by shading in the correct box.

You may use a calculator, but not one that has a graphing function, or does symbolic differentiation. Remember that all angles are assumed to be in radians.

You may not have any written aids.

1. Calculate $\lim_{x\to\infty} \frac{2x^3+3x+1}{x^2+4}$.

A. -3
B. -2
C. -1
D. 0
E. 1
F. 2
G. 3
H. $-\infty$
I. $\infty$
J. Does not exist, and is not $\infty$ or $-\infty$.

$$= \lim_{x\to\infty} \frac{2x + \frac{3}{x} + \frac{1}{x^2}}{1 + 4/x^2} = \infty$$

2. Let $h$ be the number of horizontal asymptotes, and $v$ the number of vertical asymptotes, of the graph of

$$y = \frac{x^3 - 2x^2 + 7}{(x-4)(x^2+4)}.$$

(If the function has the same limit at $-\infty$ and $+\infty$, we will count that as just one horizontal asymptote). Then $v + 3h$ is:

A. 0
B. 1
C. 2
D. 3
E. 4
F. 5
G. 6
H. 7
I. 8
J. 9

$$\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{1 - 2/x + 7/x^3}{(1-\frac{4}{x})(1+\frac{4}{x^2})} = 1$$

$\lim_{x\to-\infty} y = +1$ also

$\therefore h = 1$

As $x \to 4$, $x^3 - 2x^2 + 7 \to 39 \neq 0$

$\therefore$ There is a vertical asymptote at 4

$\therefore v = 1$

3. Evaluate $.23[1 + .21 + (.21)^2 + (.21)^3 + \ldots]$.

A. .290
B. .291
C. .292
D. .293
E. .294
F. .295
G. .296
H. .297
I. .298
J. .299

$$.23 \lim_{N \to \infty} \frac{1 - (.21)^N}{1 - .21} = .2911$$

4. Let

$$a_n = 4 + \frac{(-n)^3}{n^3 + 1}, \quad b_n = 4 + \frac{(-n)^2}{n^3 + 1}, \quad c_n = 4 + \frac{(-n)}{n^3 + 1}.$$

Consider the assertions
Consider the assertions:
(I) $\lim_{n\to\infty} a_n$ exists.
(II) $\lim_{n\to\infty} b_n$ exists.
(III) $\lim_{n\to\infty} c_n$ exists.

A. All 3 are true.
B. (I) and (II) are true, (III) is false.
C. (I) and (III) are true, (II) is false.
D. (II) and (III) are true, (I) is false.
E. (I) is true, (II) and (III) are false.
F. (II) is true, (I) and (III) are false.
G. (III) is true, (I) and (II) are false.
H. All three are false.

$$\lim_{n\to\infty} a_n = 4 + \lim_{n\to\infty} \frac{-1}{1 + 1/n^3} = 3$$

$$\lim_{n\to\infty} b_n = 4 + \lim_{n\to\infty} \frac{1}{n + \frac{1}{n^2}} = 4$$

$$\lim_{n\to\infty} c_n = 4 + \lim_{n\to\infty} \frac{-1}{n^2 + \frac{1}{n}} = 4$$

5. Calculate $\lim_{n\to\infty}(1-\frac{\pi}{n})^n$.

A. .0430
B. .0431
C. .0432
D. .0433
E. .0434
F. .0435
G. .0436
H. .0437
I. .0438
J. .0439

$e^x = \lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n$

$\therefore$ limit is $e^{-\pi} = .04321$

6. If the annual interest rate is 7%, but it is compounded monthly, how much will \$1,000 grow to after 3 years?

A. \$1,230
B. \$1,231
C. \$1,232
D. \$1,233
E. \$1,234
F. \$1,235
G. \$1,236
H. \$1,237
I. \$1,238
J. \$1,239

$1000\left(1+\frac{.07}{12}\right)^{12\times 3} = \$1232.92$

7. The half life of Carbon-14 is 5700 years. If a wood sample has 35% of the Carbon-14 that a fresh sample would have, how old is it, in years?

A. 8600
B. 8610
C. 8620
D. 8630
E. 8640
F. 8650
G. 8660
H. 8670
I. 8680
J. 8690

$A(t) = A(0)e^{-\lambda t}$

$\frac{1}{2} = e^{-\lambda 5700} \Rightarrow 5700\lambda = \ln 2$

$\therefore \lambda = \frac{\ln 2}{5700}$

If $A(t) = .35A(0)$

$e^{-\lambda t} = .35 \Rightarrow -\lambda t = \ln(.35)$

$t = \frac{-\ln(.35)}{\lambda} = \frac{-\ln(.35)}{\ln 2} \times 5700$

$= 8633$

8. Let $p(x) = x^3 - 2x^2 + 4x - 5$. Calculate $p'(1)$.

A. -5
B. -4
C. -3
D. -2
E. -1
F. 0
G. 1
H. 2
I. 3
J. 4

$p'(x) = 3x^2 - 4x + 4$

$p'(1) = 3 - 4 + 4 = 3$

9. let $f(x) = 0.5\sin(x)$. Calculate $f'(1)$.

A. -.4
B. -.3
C. -.2
D. -.1
E. 0
F. .1
G. .2
H. .3
I. .4
J. .5

$f'(x) = .5\cos x$

$f'(1) = .270$

10. Let $g(x) = \frac{x^2-2x+1}{4x-3}$. Calculate $g'(2)$.

A. .20
B. .21
C. .22
D. .23
E. .24
F. .25
G. .26
H. .27
I. .28
J. .29

$g'(x) = \frac{(4x-3)(2x-2)-(x^2-2x+1)(4)}{(4x-3)^2}$

$= \frac{4x^2-6x+2}{(4x-3)^2}$

$g'(2) = \frac{+6}{25} = .24$

11. Let $h(x) = \sqrt{\sin(2x) + e^x}$. What is $h'(\pi)$?

A. 2.1
B. 2.2
C. 2.3
D. 2.4
E. 2.5
F. 2.6
G. 2.7
H. 2.8
I. 2.9
J. 3.0

$$h'(x) = \frac{1}{2}\left[\sin(2x) + e^x\right]^{-1/2}\left[2\cos 2x + e^x\right]$$

$$h'(\pi) = \frac{1}{2}\left[e^\pi\right]^{-1/2}\left[2 + e^\pi\right]$$

$$= 2.613$$

12. Let $f(x) = (x\cos x)^4$. What is $f'(1)$?

A. -.4
B. -.3
C. -.2
D. -.1
E. 0
F. .1
G. .2
H. .3
I. .4
J. .5

$$f'(x) = 4(x\cos x)^3 \cdot \left[x(-\sin x) + \cos x\right]$$

$$f'(1) = 4\cos^3(1)\left[-\sin 1 + \cos 1\right]$$

$$= -.190$$

13. If an investment has a nominal return of 8.5% and inflation is 4.2%, what is the real rate of return?

A. 4.0%
B. 4.1%
C. 4.2%
D. 4.3%
E. 4.4%
F. 4.5%
G. 4.6%
H. 4.7%
I. 4.8%
J. 4.9%

$$\frac{(1.085)}{(1.042)} = 1.0412$$

14. Let $g(x) = \ln(\ln(1.1x))$. What is $g'(5)$?

A. .10
B. .11
C. .12
D. .13
E. .14
F. .15
G. .16
H. .17
I. .18
J. .19

$$g'(x) = \frac{1}{\ln(1.1x)} \cdot \frac{1}{1.1x} \cdot 1.1$$

$$g'(5) = \frac{1}{\ln(5.5)} \cdot \frac{1}{5} = .117$$

15. Let $f, g$ and $h$ be differentiable functions from $\mathbb{R}$ to $\mathbb{R}$. Here is a table of values for the functions and their derivatives:

| $x$ | $f(x)$ | $g(x)$ | $h(x)$ | $f'(x)$ | $g'(x)$ | $h'(x)$ |
|---|---|---|---|---|---|---|
| 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 |
| 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.0 |
| 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.0 | 1.1 |
| 1.3 | 1.4 | 1.5 | 1.6 | 1.0 | 1.1 | 1.2 |
| 1.4 | 1.5 | 1.6 | 1.0 | 1.1 | 1.2 | 1.3 |
| 1.5 | 1.6 | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 |
| 1.6 | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 |

What is the derivative of $f \circ g \circ h$ at the point 1?

A. 1.7
B. 1.8
~~C.~~ 2.0
D. 2.1
~~E.~~ 2.2
F. 2.3
G. 2.5
H. 2.6
I. 2.9
J. 3.3

$$f'(g\circ h(1))\, g'(h(1))\, h'(1)$$

$$= f'(1.5)\, g'(1.3)\, h'(1)$$

$$= (1.2)(1.1)(1.6)$$

$$= 2.112$$

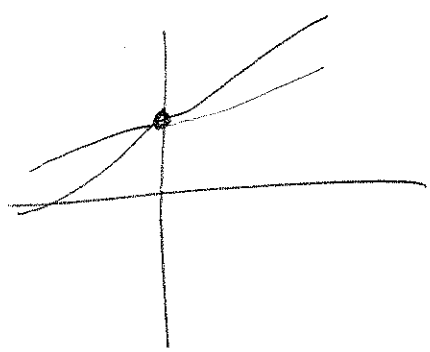
16. Let $f$ and $g$ be differentiable functions from $\mathbb{R}$ to $\mathbb{R}$. Suppose $f(0) = g(0)$ and $f'(x) \geq g'(x)$ for every $x$. Which of the following statements must be true:

(I) $f(1) \geq g(1)$.
(II) $f(-1) \leq g(-1)$.
(III) $f'(1) \geq g'(1)$.

A. All 3 are true.
B. (I) and (II) are true, (III) is false.
C. (I) and (III) are true, (II) is false.
D. (II) and (III) are true, (I) is false.
E. (I) is true, (II) and (III) are false.
F. (II) is true, (I) and (III) are false.
G. (III) is true, (I) and (II) are false.
H. All three are false.

$\frac{d}{dx}(f-g)(x) \geq 0$

$\therefore$ $f-g$ is always increasing



$\therefore$ I & II are true

III is true

Math 131 Hand-graded Problems

Student Name:

Student ID:

Problem A: Give a formal definition of the statement

$$\lim_{n\to\infty} a_n = L.$$

For every $\varepsilon > 0$, there exists $N$ such that if $n > N$, then $|a_n - L| < \varepsilon$.

Problem B: Calculate $\lim_{x\to\infty}[\sqrt{x^2+5x+1} - x]$. Show your work.

$$\sqrt{x^2+5x+1} - x = \frac{(\sqrt{x^2+5x+1} - x)(\sqrt{x^2+5x+1} + x)}{\sqrt{x^2+5x+1} + x}$$

$$= \frac{x^2+5x+1 - x^2}{x + \sqrt{x^2+5x+1}} = \frac{5x+1}{x+\sqrt{x^2+5x+1}}$$

$$= \frac{5 + 1/x}{1 + \sqrt{1 + 5/x + \frac{1}{x^2}}}$$

$$\therefore \lim_{x\to\infty}\left[\sqrt{x^2+5x+1} - x\right] = \lim_{x\to\infty} \frac{5 + \frac{1}{x}}{1+\sqrt{1+5/x+\frac{1}{x^2}}}$$

As $\lim_{x\to\infty} \frac{1}{x} = 0 = \lim_{x\to\infty} \frac{1}{x^2}$, this last limit is $\frac{5}{2}$.

$$\therefore \lim_{x\to\infty}\left[\sqrt{x^2+5x+1} - x\right] = 5/2.$$