

Math 131, Final Exam, 3.30-5.30 December 12th

This exam should have 16 multiple choice questions, and two hand-written questions. Each multiple-choice question is worth 5 points; each hand-written problem is worth 10 points.

Mark your ID number on the six blank lines on the top of your answer card, using one line for each digit. Print your name on the top of the card.

Write your name and student ID number on each of the hand-graded sheets.

Mark your answer card with a **PENCIL** by shading in the correct box.

You may not use a calculator. There is a table of functions on page 2.

You may not have any written aids.

Table of functions

x	$\sin x$	$\cos x$	$\tan x$	$\ln x$	e^x
0	0	1.0	0		1.0
0.1	0.100	0.995	0.100	-2.303	1.105
0.2	0.199	0.980	0.203	-1.609	1.221
0.3	0.296	0.955	0.309	-1.204	1.350
0.4	0.389	0.921	0.423	-0.916	1.492
0.5	0.479	0.878	0.546	-0.693	1.649
0.6	0.565	0.825	0.684	-0.511	1.822
0.7	0.644	0.765	0.842	-0.357	2.014
0.8	0.717	0.697	1.030	-0.223	2.226
0.9	0.783	0.622	1.260	-0.105	2.460
1	0.841	0.540	1.557	0.000	2.718
1.1	0.891	0.454	1.965	0.095	3.004
1.2	0.932	0.362	2.572	0.182	3.320
1.3	0.964	0.267	3.602	0.262	3.669
1.4	0.985	0.170	5.798	0.336	4.055
1.5	0.997	0.071	14.101	0.405	4.482
1.6	1.000	-0.029	-34.233	0.470	4.953
1.7	0.992	-0.129	-7.697	0.531	5.474
1.8	0.974	-0.227	-4.286	0.588	6.050
1.9	0.946	-0.323	-2.927	0.642	6.686
2	0.909	-0.416	-2.185	0.693	7.389
2.1	0.863	-0.505	-1.710	0.742	8.166
2.2	0.808	-0.589	-1.374	0.788	9.025
2.3	0.746	-0.666	-1.119	0.833	9.974
2.4	0.675	-0.737	-0.916	0.875	11.023
2.5	0.598	-0.801	-0.747	0.916	12.182
2.6	0.516	-0.857	-0.602	0.956	13.464
2.7	0.427	-0.904	-0.473	0.993	14.880
2.8	0.335	-0.942	-0.356	1.030	16.445
2.9	0.239	-0.971	-0.246	1.065	18.174
3	0.141	-0.990	-0.143	1.099	20.086
3.1	0.042	-0.999	-0.042	1.131	22.198

1. Find the minimum value of

$$f(x) = x^3 - 6x^2 + 9x + 1$$

on the interval $[2, 4]$.

A. 0

B. 1

C. 2

D. 3

E. 4

F. 5

G. 6

H. 7

I. 8

J. 9

$$\begin{aligned}f'(x) &= 3x^2 - 12x + 9 \\&= 3(x^2 - 4x + 3) \\&= 3(x-3)(x-1)\end{aligned}$$

So critical point at 3.

$$f(2) = 8 - 24 + 18 + 1 = 3$$

$$f(3) = 27 - 54 + 27 + 1 = 1$$

$$f(4) = 64 - 96 + 36 + 1 = 5$$

2. Find the maximum value of the function f given in Q. 1 on the interval $[2, 4]$.

A. 0

B. 1

C. 2

D. 3

E. 4

F. 5

G. 6

H. 7

I. 8

J. 9

3. What is the anti-derivative of $2 \cosh(2x)$?

- A. $-\frac{1}{4} \sinh(2x) + C$
- B. $\frac{1}{4} \sinh(2x) + C$
- C. $-\frac{1}{2} \sinh(2x) + C$
- D. $\frac{1}{2} \sinh(2x) + C$
- E. $-\sinh(2x) + C$
- F. $\sinh(2x) + C$
- G. $-2 \sinh(2x) + C$
- H. $2 \sinh(2x) + C$
- I. $-4 \sinh(2x) + C$
- J. $4 \sinh(2x) + C$

$$\frac{d}{dx} \cosh(2x) = \sinh(2x) \cdot 2$$

$$\frac{d}{dx} \sinh 2x = 2 \cosh(2x)$$

$\therefore \sinh 2x + C$ is antiderivative

4. Find

$$\lim_{x \rightarrow 0} \frac{\tan 4x}{\sin 2x - x}.$$

- A. ∞
- B. $-\infty$
- C. Does not exist
- D. -4
- E. -2
- F. -1
- G. 0
- H. 1
- I. 2
- J. 4

As $x \rightarrow 0$, both numerator & denominator tend to 0.

L'Hôpital \Rightarrow

$$\lim = \lim_{x \rightarrow 0} \frac{4 \sec^2 4x}{2 \cos 2x - 1} = \frac{4}{2^{-1}} = 4$$

5. Find

$$\lim_{x \rightarrow \pi^-} (x - \pi) \csc x.$$

- A. ∞
- B. $-\infty$
- C. Does not exist
- D. $-\pi$
- E. -2
- F. 1
- G. 0
- H. 1
- I. 2
- J. π

Indeterminate Product

$$\text{But} = \lim_{x \rightarrow \pi^-} \frac{x - \pi}{\sin x} \stackrel{0/0}{=} \lim_{x \rightarrow \pi^-} \frac{1}{\cos x} = -1$$

6. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a twice differentiable function. Consider the three statements:

(I) If $f'(a) = 0$, then a is a critical point.

(II) If $f'(b) = 0$, then b is a local extremum.

(III) If $f''(c) = 0$, then c is a point of inflection.

Then:

- A. All 3 are true.
- B. (I) and (II) are true, (III) is false.
- C. (I) and (III) are true, (II) is false.
- D. (II) and (III) are true, (I) is false.
- E. (I) is true, (II) and (III) are false.
- F. (II) is true, (I) and (III) are false.
- G. (III) is true, (I) and (II) are false.
- H. All three are false.

7. Let $f : (a, b) \rightarrow \mathbb{R}$ be a twice differentiable function. Assume that for all x in the interval (a, b) ,

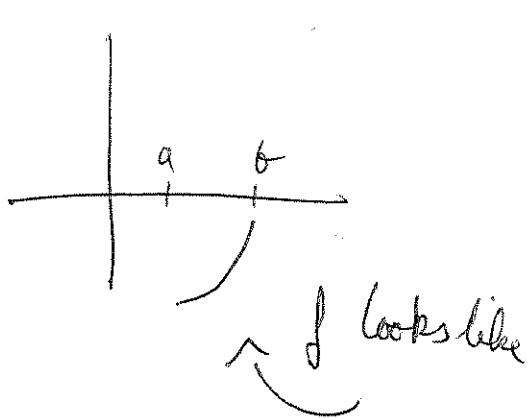
$$f(x) < 0, \quad f'(x) > 0, \quad f''(x) > 0.$$

Consider the three statements:

- (I) The function $f(x)$ is increasing on (a, b) . T
- (II) The function $[f(x)]^2$ is increasing on (a, b) . F
- (III) The graph of f is concave up on (a, b) . T

Then:

- A. All 3 are true.
- B. (I) and (II) are true, (III) is false.
- C. (I) and (III) are true, (II) is false.
- D. (II) and (III) are true, (I) is false.
- E. (I) is true, (II) and (III) are false.
- F. (II) is true, (I) and (III) are false.
- G. (III) is true, (I) and (II) are false.
- H. All three are false.



So f^2 :



8. To the nearest integer, what is

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1.5}{n}\right)^{2n}?$$

- A. 0
- B. 1
- C. 2
- D. 3
- E. 4
- F. 5
- G. 7
- H. 12
- I. 20
- J. ∞

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1.5}{n}\right)^n = e^{1.5}$$

$$\lim_{n \rightarrow \infty} \left[\left(1 + \frac{1.5}{n}\right)^n \right]^2 = e^3 = 20.086$$

9. Use the tangent line approximation to estimate $\ln(2.01)$. Which of these answers is closest?

- A. 0.691
- B. 0.692
- C. 0.693
- D. 0.694
- E. 0.695
- F. 0.696
- G. 0.697
- H. 0.698
- I. 0.699
- J. 0.700

$$\begin{aligned}\ln(2.01) &\approx \ln(2) + .01 \frac{d}{dx} \ln(x)|_2 \\ &= .693 + (.01) \frac{1}{2} \\ &= .693 + .005 \\ &= .698\end{aligned}$$

10. Evaluate the derivative of $4x^4 - 3x^3 + 4x^2 - 4$ at $x = 1$.

- A. -5
- B. -4
- C. -3
- D. -2
- E. -1
- F. 0
- G. 1
- H. 2
- I. 3
- J. 4

$$4x^3 - 9x^2 + 8x|_1 = 9$$

11. The half-life of a certain substance is 693 seconds. After 1000 seconds, 1 kg of the substance remains. How many kg were there initially, to one decimal place?

- A. 2.1
- B. 2.2
- C. 2.3
- D. 2.4
- E. 2.5
- F. 2.6
- G. 2.7
- H. 2.8
- I. 2.9
- J. 3.0

$$A(t) = A_0 e^{-kt}$$

$$A(1000) = A_0 e^{-1000k} = 1$$

$$A(693) = A_0 e^{-693k} = \frac{1}{2} A_0$$

$$e^{-693k} = \frac{1}{2}$$

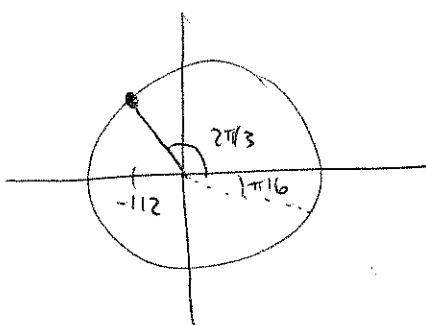
$$-693k = \ln(\frac{1}{2}) = -\ln 2 \\ = 693$$

12. If $\arccos(x) = \frac{2\pi}{3}$, what is $\arcsin(x)$?

- A. $-\frac{5\pi}{6}$
- B. $-\frac{2\pi}{3}$
- C. $-\frac{\pi}{2}$
- D. $-\frac{\pi}{3}$
- E. $-\frac{\pi}{6}$
- F. $\frac{\pi}{6}$
- G. $\frac{\pi}{3}$
- H. $\frac{\pi}{2}$
- I. $\frac{2\pi}{3}$
- J. $\frac{5\pi}{6}$

$$x = \cos(\frac{2\pi}{3})$$

$$= -\frac{1}{2}$$



$$\therefore k = 1/693$$

$$\therefore A(1000) = A_0 e^{-1000k} = 1 \\ \therefore A_0 = e^{1000k} = e^{1000/693} \approx 2.7$$

$$\therefore \arcsin(x)$$

$$= \arcsin(-\frac{1}{2})$$

$$= -\pi/6$$

13. What is $\lim_{x \rightarrow \infty} e^{2/x}$?

- A. $-\infty$
- B. -2
- C. -1
- D. 0
- E. $1/2$
- F. 1
- G. 2
- H. 4
- I. ∞
- J. Does not exist.

$$\lim_{x \rightarrow \infty} \frac{2}{x} = 0$$
$$\therefore \lim_{x \rightarrow \infty} e^{2/x} = e^0 = 1$$

14. Differentiate $\sin^2(x^2)$.

- A. $2 \cos(x^2)$
- B. $4 \cos(x^2)$
- C. $2x \cos(x^2)$
- D. $4x \cos(x^2)$
- E. $2 \sin(x^2) \cos(x^2)$
- F. $4 \sin(x^2) \cos(x^2)$
- G. $2x \sin(x^2) \cos(x^2)$
- H. $4x \sin(x^2) \cos(x^2)$
- I. $2x^2 \sin(x^2) \cos(x^2)$
- J. $4x^2 \sin(x^2) \cos(x^2)$

$$2 \sin(x^2) \cdot \cos(x^2) \cdot 2x$$

15. What is the slope of the line tangent to the curve given by $x^3 + 3x^2y^2 + 5y^3 + y = 8$ at the point $(2, 0)$?

- A. 4
- B. -4
- C. 8
- D. -8
- E. 10
- F. -10
- G. 12
- H. -12
- I. 14
- J. -14

$$3x^2 + 6xy^2 + 3x^2 \cdot 2y \frac{dy}{dx} + 15y^2 \frac{dy}{dx} + \frac{dy}{dx} = 0.$$

At $(2, 0)$:

$$12 + \frac{dy}{dx} = 0$$

16. If $f(x) = x^4 + 2x^3 - 1$, what is $(f^{-1})'(2)$?
(Note that $f(1) = 2$).

- A. 1
- B. 1/2
- C. 1/3
- D. 1/4
- E. 1/5
- F. 1/6
- G. 1/7
- H. 1/8
- I. 1/9
- J. 1/10

$$f'(x) = 4x^3 + 6x^2$$

$$f'(1) = 10$$

$$\therefore f^{-1}'(f(1)) = \frac{1}{10}$$

Math 131 Hand-graded Problems

Student Name:

Student ID:

Problem A: Give a rigorous definition of what it means to say

$$\lim_{x \rightarrow a} f(x) = L.$$

for every $\epsilon > 0$ there exists $\delta > 0$ such that
 if $0 < |x - a| < \delta$ then $|f(x) - L| < \epsilon$.

Problem B: Find two positive integers so that the sum of the first number and 5 times the second number is 2000 and the product of the two numbers is as large as possible.

Constraint $x + 5y = 2000 \quad (C)$

Maximize xy .

From (C), $x = 2000 - 5y$

Maximize $(2000 - 5y)y = p(y)$ on $[0, 200]$

$$p(y) = 2000y - 5y^2$$

Critical point at $y = 200$

$$p'(y) = 2000 - 10y$$

$$\text{Maximum} = p(200)$$

$$= 200(1000)$$

Answer: 100 and 200.