Math 100: Foundations for Calculus

Exam 3

October 28, 2002

Please put your name on this sheet and on each piece of lined paper that you use. Show your work and write your answers on the lined paper, not on this sheet (except for problems 4b and 11). Be sure to follow directions carefully and show all your work. You may not use a graphing calculator. There is a total of 40 points on the exam.

(6) 1. Carefully graph the following function. Be sure to mark a scale on each axis.

\[ f(x) = \begin{cases} 
  x^3 & \text{if } x < 1 \\
  3 - x & \text{if } x \geq 1
\end{cases} \]

(2) 2. Consider the function whose graph is shown to the right.

(a) What is the domain of this function?
(b) What is the range of this function?

You may write your answers either in inequality notation or in interval notation.

(3) 3. Find the average rate of change of the function \( f(x) = 2x^2 \) between \( x = -1 \) and \( x = 3 \).

(4) 4. Consider the function \( f \) whose graph is shown below, and consider the function \( g \) which is given by \( g(x) = -2f(x - 1) \).

(a) Express in words what transformations have been done to \( f \) to obtain \( g \).
(b) Draw the graph of \( g \). (If you wish, you may draw it above along with \( f \).)

(5) 5. Consider the quadratic function \( f(x) = -3x^2 - 30x + 8 \).

(a) Put the function in standard form by completing the square.
(b) What is the vertex of this parabola?

(2) 6. Let \( f(x) = x^2 - 2x \) and let \( g(x) = \frac{3}{x} \). Find \( (f \circ g)(x) \). You do not need to simplify.
(2) 7. Consider the functions $f$ and $g$ whose graphs are shown below.

(a) Find $(f + g)(1)$.
(b) Find $(g \circ f)(2)$.

(2) 8. Let $h(x) = \sqrt{x - 5}$. Find functions $f$ and $g$ such that $h(x) = (g \circ f)(x)$.

(2) 9. (a) Which one of the four graphs below portrays a one-to-one function?
(b) Which one of the four graphs below portrays something which is a function but is not one-to-one?

(5) 10. The function $f(x) = \frac{-x}{2x+5}$ is one-to-one. Find its inverse function.

(2) 11. Consider the one-to-one function $f$ whose graph is shown below. Draw the graph of its inverse function. (If you wish, you may draw it below along with $f$.)

(5) 12. Carrie wishes to lay out a garden with a wall on all four sides, with the back wall higher than the other three walls. The material for the back wall will cost $15 per foot, but the material for the other three sides will cost only $5 per foot. Carrie has a total of $1000 to spend on the material for the walls, and she would like to know what dimensions will give her the greatest area for her garden. Find a function that models the area of the garden in terms of either its length or its width. You do not need to continue the problem to the point where you could answer Carrie's question.