Please follow directions carefully and show all your work. Show your work and write your answers on the lined paper or graph paper, not on this sheet. Put your name on each page that you use. There is a total of 40 points on the exam.

Part I. (32 points) You may not use a graphing calculator on this part of the exam.

(4) 1. (a) Write out the definitions of the six trigonometric functions in terms of $x$, $y$, and $r$.

(b) Explain what $x$, $y$, and $r$ represent. You may do this in words or by showing them on the picture to the right.

(4) 2. (a) Draw the 30–60–90 triangle and label its angles and sides.

(b) Draw the 45–45–90 triangle and label its angles and sides.

(6) 3. Find each of the following.

(a) $\csc 30^\circ$

(b) $\cos(-\pi)$

(c) $\cot 150^\circ$

(3) 4. For each of the following questions about the trigonometric functions, select your answer from the choices below.

(a) Which two of the six trigonometric functions are the only ones whose graphs are continuous?

(b) Which two of the six trigonometric functions are the only ones which are positive for all angles with terminal side in quadrant II?

(c) Which two of the six trigonometric functions are the only ones whose graphs are symmetric with respect to the $y$-axis (and thus are even functions)?

(I) $y = \sin x$ and $y = \cos x$

(II) $y = \tan x$ and $y = \cot x$

(III) $y = \sec x$ and $y = \csc x$

(IV) $y = \sin x$ and $y = \csc x$

(V) $y = \cos x$ and $y = \sec x$
(2) 5. Complete each of the following identities.

(a) \( \frac{\cos \theta}{\sin \theta} = \) 

(b) \( \tan^2 \theta + 1 = \) 

(5) 6. (a) Graph the function \( y = -\frac{1}{2} \sin 4x \).

(b) What is the amplitude of this function?

(c) What is the period of this function?

(1) 7. Briefly answer one or the other of the following questions, but not both.

(a) Why is it necessary to restrict the domains of the trigonometric functions before we can define the inverse trigonometric functions? In other words, what is the problem with the trig functions?

(b) What is the difference between \( \sin^{-1} \theta \) and \( (\sin \theta)^{-1} \)?

(4) 8. Find each of the following.

(a) \( \sin^{-1} \left( \frac{1}{\sqrt{2}} \right) \)

(b) \( \tan(\sec^{-1} \left( \frac{10}{3} \right)) \)

(3) 9. Use the formula \( \cos(x + y) = \cos x \cdot \cos y - \sin x \cdot \sin y \) to find the value of \( \cos 75^\circ \). You do not need to simplify your answer.
Part II. (8 points) You may use a graphing calculator on this part of the exam.

(6) 10. A sample of cesium–137 decays according to the equation \( m(t) = 100e^{-0.0231t} \), where \( t \) is time in years and \( m(t) \) is the mass of the sample in grams at time \( t \).

(a) What is the initial amount of the sample?

(b) How long does it take for the sample to decay to 30 grams? Round your answer to the nearest year.

(2) 11. Evaluate each of the following.

(a) \( \tan(-53^\circ) \)

(b) \( \sec\left(\frac{\pi}{3}\right) \)