MATH 127, FALL 2003
-EXAM 1-

PART I consists of 14 multiple choice questions (worth 5 points each), for a total of 70 points, and PART II consists of 6 true/false questions (worth 1 point each), for a total of 6 points. Mark the correct answer on the answer card. On PARTs I and II, only the answer on the card will be graded.

PART III consists of 3 questions (worth 8 points each), for a total of 24 points.

No calculators with a CAS are allowed.
PART I - Blacken your answers on the answer card.

1. For \( f(x) = \frac{x - 2}{x - 1} \) and \( g(x) = \frac{x^2 + 4}{3} \), find \( \frac{f(x)}{g(x)} \).

   (A) \( \frac{3x - 6}{(x - 1)(x^2 + 4)} \)  \( \quad \) (B) \( \frac{3}{(x - 1)(x + 2)} \)  \( \quad \) (C) \( \frac{(x - 2)(x^2 + 4)}{3x - 3} \)

   (D) \( \frac{3(x - 2)(x^2 + 4)}{x - 1} \)  \( \quad \) (E) \( \frac{(x - 2)(x^2 + 4)}{x - 1} \)  \( \quad \) (F) \( \frac{x^2 + 4}{3x - 3} \)

   (G) \( \frac{x + 2}{3x - 3} \)  \( \quad \) (H) \( \frac{(x - 2)(x^2 + 4)}{3} \)  \( \quad \) (I) \( \frac{(x - 1)(x - 2)(x^2 + 4)}{3} \)

\[
\frac{x - 2}{x - 1}, \quad \frac{3}{x^2 + 4} = \frac{3x - 6}{(x - 1)(x^2 + 4)}
\]
2. Which of the following graphs is a graph of a function?

A) A  B) B  C) C  D) D

H) A,B,C  I) B,C,D


The: vertical line test
3. Which of the following functions have domains not equal to all real numbers?

A) \( f(x) = \sqrt{x^2 + 4} \)   B) \( f(x) = x^\frac{3}{2} \)   C) \( f(x) = \frac{1}{e^x} \)   D) \( f(x) = \frac{2x}{41 - x} \)

A) A,B \quad B) A,C \quad C) A,D \quad D) B,C \quad E) B,D \quad F) A,B,C \quad G) A,C,D \quad H) B,C,D \quad I) A,B,C,D

B. \( f(x) = \sqrt{x^3} \) \quad \text{domain: } \{0, \infty\}

D. \( f(x) = \frac{2x}{41 - x} \) \quad \text{domain: } (-\infty, 41) \cup (41, \infty)
4. For \( f(x) = .5 + 30x \), find \( \frac{f(a + h) - f(a)}{h} \).

A) \(.5 + 30h\)   B) \(\frac{.5 + 30x}{x}\)   C) \(\frac{.5 + 30h}{x}\)   D) 30   E) \(\frac{30}{h}\)
F) .5   G) \(\frac{5}{h}\)   H) -30   I) -.5

\[
\begin{align*}
f(a+h) &= .5 + 30(a+h) = .5 + 30a + 30h \\
f(a) &= .5 + 30a
\end{align*}
\]

\[
\therefore f(a+h) - f(a) = 30h
\]

\[
\therefore \frac{f(a+h) - f(a)}{h} = \frac{30h}{h} = 30 \quad \text{if} \ h \neq 0
\]

\(\boxed{D}\)
5. An art studio charges $80 for a series of four two hour art lessons. The studio's costs are $15 per lesson for the room rental, $30 per hour for the instructor, and $3 per student for miscellaneous expenses. If \( x \) is the number of students enrolled in the class, find a function expressing the studio's profit \( P(x) \) in terms of \( x \). Find the profit if 10 students enroll in the class.

A) \( P(x)=77x-360; \ P(10)=410 \)  
B) \( P(x)=80x-192; \ P(10)=608 \)
C) \( P(x)=77x-180; \ P(10)=590 \)  
D) \( P(x)=80x-48; \ P(10)=752 \)
E) \( P(x)=77x-300; \ P(10)=470 \)  
F) \( P(x)=80x-324; \ P(10)=476 \)
G) \( P(x)=80x-45; \ P(10)=755 \)  
H) \( P(x)=47x-120; \ P(10)=350 \)
I) \( P(x)=50x-63; \ P(10)=437 \)

\[
P(x) = 80x - (15 \cdot 4 + 30 \cdot 8 + 3x) \\
= 77x - 300
\]

\[
P(10) = 770 - 300 = 470
\]

\( \therefore \) E}
6. State the domain and range of the function

\[ f(x) = \begin{cases} 
  x + 5 & \text{if } -10 \leq x \leq 5, \\
  -3x + 21 & \text{if } 5 < x \leq 22
\end{cases} \]

A) domain= \(( -\infty, \infty)\); range= \(( -\infty, \infty)\)

B) domain= \(( -10, 22)\); range= \(( -45, -5) \cup (6, 10)\)

C) domain= \(( -\infty, \infty)\); range= \(( -45, 10)\)

D) domain= \([ -10, 22]\); range= \([ -45, -5] \cup (6, 10]\)

E) domain= \([ -10, 5) \cup (5, 22]\); range= \([ -45, 10]\)

F) domain= \([ -10, 22]\); range= \([ -45, 10]\)

G) domain= \([-10, 22]\); range= \([-5, 10]\)

H) domain= \((-10, 5) \cup (5, 22]\); range= \((-45, 10)\)

I) domain= \(( -\infty, \infty)\); range= \(( -45, -5) \cup (6, 10)\)

\[ \text{domain } = \([ -10, 22]\) \]

\[ \text{range } = \([ -45, 10]\) \]
7. The management of a company that manufactures surfboards has fixed costs (at 0 output) of $200 per day and total costs of $4000 per day at a daily output of 20 boards. Assuming that the total cost per day, $C(x)$, is linearly related to the total output per day, $x$, write an equation for the cost function.

A) $C(x) = 4000 + x$
B) $C(x) = 200 + 190x$
C) $C(x) = 200 + 200x$
D) $C(x) = 4000 + 200x$
E) $C(x) = 4000x - 200$
F) $C(x) = 4000(x - 200)$
G) $C(x) = 200 + 4000x$
H) $C(x) = 200 + x$
I) $C(x) = 3800x$

Points on line:

$\begin{align*}
(x, C(x)) \\
(0, 200) \\
(20, 4000)
\end{align*}$

\[ \text{slope} = \frac{4000 - 200}{20 - 0} = 190 \]

\[ y - 200 = 190x \]

\[ y = 200 + 190x \]
8. Find an equation for the line passing through the points (2, 20) and (2, -105).

A) $x = 2$

B) $y = 2$

C) $\frac{y-20}{x-2} = 135$

D) $\frac{y-20}{x-2} = 85$

E) $y = 20$

F) $2x + 20y = 0$

G) $2x - 105y = 0$

H) undefined

I) $\frac{y}{x} = \frac{-105}{20}$
9. Find an equation for the line which passes through the point \((4, -2)\) and has slope 3.

A) \(x - y = 6\)

B) \(3x + y = 14\)

C) \(x + 3y = -2\)

D) \(3x - y = 14\)

E) \(x - 3y = 10\)

F) \(x + y = 2\)

G) \(y - x = -2\)

H) \(4x - 2y = 3\)

I) \(2x - 4y = 3\)

\[
y - (-2) = 3(x - 4)
\]

\[
y = 3x - 14
\]

\[
\therefore 3x - y = 14
\]
10. Solve the inequality $3x^2 - 12,000x > 9,000,000$.

A) $(3000, \infty)$
B) $(-\infty, 3000)$
C) $(-\infty, 3000) \cup (1000, \infty)$
D) $(1000, 3000)$
E) $(-\infty, 1000)$
F) No solution exists.

\[ \text{(G) } (-\infty, 1000) \cup (3000, \infty) \]

H) $(-\infty, -1000) \cup (3000, \infty)$
I) $(-\infty, -3000)$

First solve

\[
3x^2 - 12,000x + 9,000,000 = 0
\]

\[
3(x^2 - 4000x + 3000000) = 0
\]

\[
\therefore 3(x - 3000)(x - 1000) = 0
\]

\[
\frac{+}{-} \quad \frac{1}{1000} \quad \frac{-}{3000} \quad \frac{+}{+}
\]

\[
\therefore (-\infty, 1000) \cup (3000, \infty)
\]

\[ \text{(G)} \]
11. Suppose \( f(x) \) is a polynomial of degree 8.

(1) What is the maximum number of turning points of the graph of \( f \)?

(2) What is the maximum number of \( x \)-intercepts of the graph of \( f \)?

8 - 1 = 7

8

A) 8 B) 7; 7 C) 8; 7 (D) 7; 8 E) 8; 16 F) 1; 2 G) 2; 1
H) no maximum exists; 8 I) 8; no maximum exists
12. For the rational function \( f(x) = \frac{10^6x}{2x-2} \), find the following:
   (1) the domain
   (2) any vertical asymptotes for the graph
   (3) any horizontal asymptotes for the graph

   A) \((\infty, \infty)\); No asymptotes
   B) \((\infty, \infty)\); \(x = 1\); \(y = 10^6\)
   C) \((\infty, \infty)\); \(x = 0\); \(y = 10^6\)
   D) \((\infty, 0) \cup (0, \infty)\); \(x = 0\); \(y = 10^6/2\)
   E) \((\infty, 0) \cup (0, \infty)\); \(x = 10^6/2\); \(y = 0\)
   F) \((\infty, 0) \cup (0, \infty)\); \(x = 10^6\); \(y = 0\)
   G) \((\infty, 1) \cup (1, \infty)\); \(x = 1\); \(y = 10^6/2\)
   H) \((\infty, 1) \cup (1, \infty)\); \(x = 0\); \(y = 10^6\)
   I) \((\infty, 1) \cup (1, \infty)\); \(x = 0\); \(y = 10^6/2\)

(1) domain: all \(x\) except \(x = 1\) (when \(2x - 2 = 0 \Rightarrow \frac{10^6x}{2x-2} = \frac{10^6}{2}\))

(2) vertical asymptote at \(x = 1\)

(3) \(f(x) = \frac{10^6x}{2x-2} = \frac{10^6}{2-\frac{2}{x}}\)

\(\therefore\) as \(x \to \infty\), \(f(x) \to \frac{10^6}{2}\)

\(\therefore\) horizontal asymptote \(y = \frac{10^6}{2}\)
13. Solve for \( x \): \( 3^{9x} = 27^{x+5} \).

A) \( \frac{15}{7} \)

B) 0

C) \( \log_3 9 \)

D) \( -\log_3 5 \)

E) 3

F) \( \frac{5}{2} \)

G) \( \frac{1}{2} \)

H) \( \frac{9}{5} \)

I) 4

\[
3^{9x} = 3^{3(x+5)}
\]

\[
9x = 3x + 15
\]

\[
x = 15
\]

\[
x = \frac{15}{6} = \frac{5}{2}
\]

\( \boxed{\text{F}} \)
14. Solve for $x$: $5xe^{-x} + x^2e^{-x} = 0$.

A) 0
B) 0, e
C) 0, e, -5
D) -5
E) [-5,0]
F) 0, -5
G) (-5,0)
H) No solution exists
I) $e^{-5}$

**Factor** \[ xe^{-x}(5 + x) = 0 \]

\[ \therefore x = 0 \quad \text{or} \quad x = -5 \]
Part II. True-False (1 point each) Mark your card “A” if the statement is true and “B” if the statement is false.

15. A quadratic function always has a minimum.
   \[ F \quad \text{eg: } y = -x^2 \]

16. The polynomial \( f(x) = 3x^{10001} - 0.5x^{50} + 1.75 \) has no roots outside the interval \([-2, 2]\).
   \[
   T: \text{apply Thm 1:} \\
   1\cdot 1 \leq 1 + \max \left\{ \frac{5}{3}, \frac{1.75}{3} \right\} < 2
   \]

17. The polynomial \( f(x) = x^{4000000} + x^{30} - 3 \) must cross the \( x \)-axis.
   \[
   T: \quad f(0) = -3 \\
   f(10) > 0
   \]

18. If \( f(25) = 1 \), then 25 is a root of the function \( g(x) = f(x) - 1 \).
   \[
   T
   \]

19. The graph of a rational function can never cross one of its asymptotes.
   \[
   F: \quad \text{eg: } f(x) = \frac{x^2 - x + 100}{x^2} \\
   \text{hor. asym: } y = 1 \text{ but } f(100) = 1.00001
   \]

20. The exponential function \( f(x) = e^x \) has a vertical asymptote.
   \[
   F
   \]
   \[ \text{domain: } e^x = \text{all reals.} \]
Part III: These are three “free response” problems worth a total of 24 points. Write your answers on the test pages. Show your work neatly and cross out irrelevant scratchwork, false starts, etc.

Please put your Washington University ID number on each of the following pages as they might be separated during grading. Do NOT put your name on these pages. Also, please add your Discussion Section Letter (available on your exam front cover sheet) on each page so that we can return papers through discussion sections.

WashU ID Number: ___________ Discussion Section Letter: __

21. Write an equation for the lowest degree polynomial with the graph and intercepts shown.

0, 1, -1 = roots

\[ y = a (x - 0)(x - 1)(x - (-1)) \]

\[ = a (x^2 - x) \]

Plug in: \( x = \frac{1}{2} \).

See from graph: \( y = \frac{3}{4} \)

\[ \therefore \frac{3}{4} = y = a \left( \frac{1}{8} - \frac{1}{2} \right) = -\frac{3a}{8} \]

Solve for \( a \):

\[ a = -8, \quad \frac{3}{4} = -2 \]

\[ \therefore y = -2(x^3 - x) = -2x^3 + 2x \frac{3}{2} \]
22. For the given graph of \( y = f(x) \), graph the function defined by \( y = -2f(x - 2) - 5 \).
23. In a study on the speed of muscle contraction in frogs under various loads, researchers W. O. Fems and J. Marsh found that the speed of contraction decreases with increasing loads. In particular, they found that the relationship between speed of contraction $v$ (in centimeters per second) and load $x$ (in grams) is given approximately by

$$v(x) = \frac{26 + .08x}{x} \text{ when } x \geq 5$$

(A) What is the domain of $v$?

(B) What is the range of $v$?

(C) Does the function $v$ have a maximum? If so, what is it?

(D) Does the function $v$ have a minimum? If so, what is it?

(E) Without using your calculator, estimate $v(10^{10^{10}})$ to two decimal places. Explain your answer.

(A) domain: $v = (5, \infty)$

(B) $v$ decreasing given

$$\Rightarrow \text{ max value is } v(5) = 5.28$$

As $x \rightarrow \infty$, $v(x) = \frac{26 + .08x}{x} = \frac{26}{x} + .08 \rightarrow 0.08$

$$\Rightarrow \text{ range is } (0.08, 5.28]$$

(C) max = 5.28

(D) no min since never reaches 0.08

(E) $v(10^{10^{10}}) = \frac{26}{10^{10^{10}}} + .08 \rightarrow .08, \text{ small } < .001$