Math 128 Exam 1 September 26, 2005

This exam has 21 questions. The first 14 are multiple choice, 5 points each, the next 5 are true-false, 2 points each. The answers for those 19 questions go on the answer card. The last two questions, 10 points each, will be graded by hand. To receive full credit on those questions you must show work which justifies your answer.

1. Given

$$f(x) = x^3 \ln(2x^3 + 1).$$

Find $f'(2)$.

a. $2\ln 14 + \frac{1}{15}$

b. $2\ln 3 + \frac{2}{37}$

c. $-2\ln 3 + \frac{4}{7}$

d. $6\ln 6 + \frac{1}{15}$

e. $12\ln 17 + \frac{192}{17}$

f. $4\ln 2 + \frac{144}{23}$

g. $4\ln 14 + \frac{96}{5}$

h. $-5\ln 23 + \frac{5}{144}$

i. $-6\ln 5 + \frac{16}{15}$

j. $15\ln 3 + \frac{44}{15}$

2. Find the minimum value, if there is one, of the function defined for $x > 0$ by

$$f(x) = 3x^3 + \frac{144}{x}.$$

a. 3

b. 6

c. 12

d. 24

e. 48

f. 96

g. 122

h. 144

i. 196

j. There is no minimum.

3. For

$$f(x) = \sqrt{\frac{1}{x} + (2x+1)^2},$$

find $f'(1)$.

a. 0

b. $3\sqrt{2}$

c. $\frac{3}{4}\sqrt{12}$

d. $\frac{4}{3}\sqrt{10}$

e. $2\sqrt{5}$

f. $\frac{5}{3}\sqrt{2}$

g. $\frac{11}{20}\sqrt{10}$

h. $\frac{4}{3}\sqrt{\frac{5}{3}}$

i. $5\sqrt{\frac{3}{2}}$

j. $\frac{3}{5}\sqrt{\frac{5}{2}}$

4. Evaluate the integral

$$\int_1^2 \sqrt{2x+3}\,dx.$$

a. $\sqrt{2} - 1$

b. $\frac{7}{3}\sqrt{7} - \frac{5}{3}\sqrt{5}$

c. $\sqrt{7} + \sqrt{5}$

d. $\sqrt{3} - \sqrt{2}$

e. $\frac{1}{3}\sqrt{7} - \frac{1}{3}\sqrt{5}$

f. $2\sqrt{7} - 3\sqrt{5}$

g. $3\sqrt{2} - 3$

h. $3\sqrt{3} - 2$

i. 0

j. $2\sqrt{7} + 3\sqrt{3}$

5. Evaluate the integral

$$\int_0^1 \frac{x^2}{x^3+1}\,dx.$$

a. $3\ln 2$

b. $2\ln 3$

c. $\ln \frac{3}{2}$

d. $\frac{1}{3}\ln 2$

e. $\frac{3}{2}$

f. $\frac{2}{3}$

g. $3\sqrt{2}-3$

h. $2\sqrt{3}-2$

i. $\ln 3 - \ln 2$

j. $\sqrt{3} - 2\ln 3$

6. Find the area between the curves $y = 2x - x^2$ and $y = -x$.

a. 3/4

b. 5/4

c. 3/8

d. 5/8

e. 8/3

f. 9/2

g. 7/3

h. 4/5

i. 7/4

j. 4/7

7. The Lorentz curve for an income distribution is given by $y = x^{1.8}$. Find the associated Gini index.

   a. .14864
   b. .19323
   c. .28571
   d. .37212
   e. .43965
   f. .55932
   g. .62396
   h. .66384
   i. .70141
   j. .77125

8. A metropolitan area currently has a population of 2 million. The population is growing at 3% per year. How long will it be until the population reaches 3 million?

   a. 13.52 years
   b. 15.65 years
   c. 15.78 years
   d. 16.23 years
   e. 16.68 years
   f. 16.93 years
   g. 17.12 years
   h. 17.44 years
   i. 18.02 years
   j. 18.31 years

9. Radioactive carbon, carbon-14, decays at a rate proportional to the amount present. The rate constant is $r = -.0001238$. How many years will it take for 40% of the sample to decay?

   a. 2785.1
   b. 3021.3
   c. 3774.9
   d. 4126.2
   e. 4225.7
   f. 4895.5
   g. 5198.2
   h. 5692.6
   i. 5993.6
   j. 6239.3

The next 2 problems concern an income stream of $2000 per year for 5 years which is invested at 8% compounded continuously

10. What is the future value of the income stream 5 years from now?

   a. $11,153
   b. $11,325
   c. $11,396
   d. $11,488
   e. $11,645
   f. $11,722
   g. $11,904
   h. $12,095
   i. $12,296
   j. $12,395

11. What is the present value of that income stream?

a. $\$7,702$

b. $\$7,735$

c. $\$7,894$

d. $\$7,961$

e. $\$8,053$

f. $\$8,109$

g. $\$8,178$

h. $\$8,242$

i. $\$8,314$

j. $\$7,520$

12. What is the area between the curve $y = \ln x$ and the x–axis for $1 \leq x \leq e$

a. 0

b. 1

c. 2

d. e

e. $2e$

f. $e/2$

g. $e/2$

h. $2 - e/2$

i. $2 + e/2$

j. e^2

13. Evaluate

$$\int_1^2 x^3 \ln(4x)dx.$$

a. $\ln 8 - 2\ln 4 - \frac{5}{16}$

b. $3\ln 3 - \ln 4 - \frac{1}{4}$

c. $4\ln 8 - \frac{1}{4}\ln 4 - \frac{15}{16}$

d. $8\ln 2 - 4$

e. $8\ln 2 - 2$

f. $4\ln 2 - 4$

g. $3\ln 3 - 5\ln 4$

h. $3\ln 4 - 4$

i. $3\ln 4 - 2\ln 3 - 2$

j. $4\ln 3 - \ln 4 - 6$

14. Evaluate

$$\int_1^2 (2x+1)e^{3x}dx.$$

a. $e^6 - e^3$

b. $4e^6 - 2e^3 - 2$

c. $e^6 - e^3 - 2$

d. $2e^6 - 3e^3$

e. $2e^6 - 3e^3 + 4$

f. $\frac{7}{9}e^6 - \frac{5}{9}e^3$

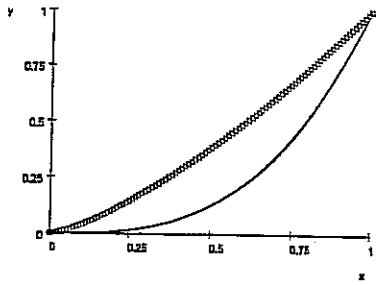
g. $\frac{1}{9}e^6 - \frac{4}{9}e^3$

h. $\frac{4}{7}e^6 - \frac{2}{7}e^3$

i. $\frac{3}{4}e^6 - \frac{8}{9}e^3$

j. $\frac{13}{9}e^6 - \frac{7}{9}e^3$

The next five questions are true-false questions. The first two refer the Lorentz curves for distribution of family income shown in the following figure.



15. On the two Lorentz curves the lower, lighter, curve corresponds to a more even distribution of income.

a. True
b. False

16. The Gini index corresponding to the lower, lighter curve is smaller than that for the higher, heavier curve.

a. True
b. False

17. A population is growing at 3% a year. Using the rule of 72 we can estimate that the population will be twice as large in 24 years.

a. True
b. False

18. If a quantity is changing at the rate $r(t)$ at time t then the total change from time $t = a$ to time $t = b$ is given by $\int_a^b r(t)dt$.

a. True
b. False

19. (Assuming that interest rates are positive) the future value on an income stream is always less then the present value of the same income stream.

a. True
b. False

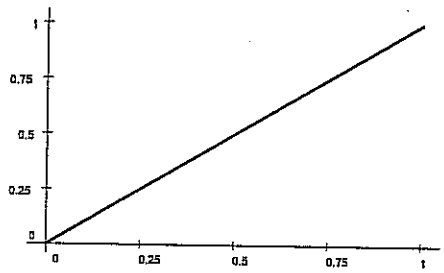
Name:_____ ID Number:_____

This question will be hand graded, to get full credit in needs to be clear HOW you arrived at your answer.

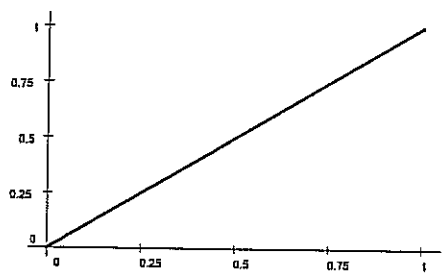
20. Here is the data on how much of the family income goes to five different groups in a country.

Bottom 20%	Next 20%	Mid 20%	Next 20%	Top 20%
3%	7%	15%	30%	45%

a. On the given grid, which already has the line $x = y$ included for your convenience, sketch the Lorentz Curve for the data. Indicate the coordinates of the points you plot, either on the graph or in a table.



b. Explain briefly but clearly, using a picture if it is helpful, what a meant by a right sum for estimating the area between the Lorentz curve.



c. In fact the right sum for this data is .386 and the left sum is .186. Using these numbers compute the Gini index associated with this curve.

Name:_____ ID Number:_____

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21. Evaluate the following indefinite integrals. Either show enough work or give enough discussion so that it is clear how you got your answer.

a. $\int e^x dx$

b. $\int xe^x dx$

c. $\int xe^{x^2} dx$

d. $\int x^2 e^x dx$