

Math 128

Midterm Examination 1 – September 23, 2008

Name _____

6 problems, 100 points.

Instructions: Show all work – partial credit will be given, and “Answers without work are worth credit without points.” You don’t have to simplify your answers. You may use a simple calculator that is not graphing or programmable. You may have a 3x5 card, but no other notes.

1. (18 points) Compute the following partial derivatives

(a) h_z , where $h(x, y, z) = \frac{x^2 + y^2 + z^2}{xyz}$

(b) $\frac{\partial}{\partial x} \ln(e^{x^2y} + e^{y^2x})$

(c) f_{xy} , where $f(x, y) = 3 \sin xy$

2. (12 points) Sketch the level curves of $z = y - x^2$ at the levels $z = 0, 1, 2$.
Make sure to label your graph.

3. (20 points) Let $f(x, y) = x^2 + 4y^3 - 6xy + 10$.

(a) Find the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

(b) Find the critical points for f

(c) Calculate the 2nd derivatives $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial y^2}$, and $\frac{\partial^2 f}{\partial x \partial y}$.

(d) Using the 2nd derivative test, determine which points are relative maxima, relative minima, and saddle points.

4. A small company has a local monopoly on two competing products: model houses, and model hotels. It costs them \$200 to build each model house, \$300 for each hotel. The total revenue from selling x houses and y hotels is $1000x + 1200y - 2xy - x^2 - 2y^2$.

(a) (5 points) What is the *profit* $P(x, y)$ from selling x houses and y hotels?

(b) (5 points) Calculate the partial derivatives P_x and P_y .

(c) (5 points) Find the number of houses and hotels the company should build to maximize their profit.

(d) (3 points) Explain briefly why your answer in (c) is a maximum.

5. Consider the function $z = 3x + y$ on the ellipse $4x^2 + y^2 = 25$.
- (a) (5 points) Set up a Lagrange multiplier function $F(x, y, \lambda)$ for z subject to this constraint.
- (b) (12 points) Find all critical points for F .
- (c) (5 points) Determine the maximum and minimum value of z on the ellipse $4x^2 + y^2 = 25$.

6. (10 points) Consider the function $z = f(x, y) = x^2 - xy + y^2$.

(a) What is the slope of the tangent line to f in the cross section $x = 1$ at the point $(1, 2, 3)$?

(b) The real number $f_x(2, 3) = \frac{\partial f}{\partial x}|_{x=2, y=3}$ is the slope of a tangent line to f at some point, in some cross sectional plane. What is the point, and what is the equation of the cross section plane?