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This exam has 14 questions:

- 12 multiple choice worth 5 points each.
- 2 hand graded worth 40 points total.

Important:

- No graphing calculators!
- For the multiple choice questions, mark your answer on the answer card.
- Show all your work for the written problems. You will be graded on the ease of reading your solution.
- You are allowed a 3×5 note card for the exam.

1. Which of the following best describes the domain of the function $f(x, y) = \ln(x^2 + y^2 - 1)$?

   (a) All of the xy-plane
   (b) All of the xy-plane except the origin
   (c) All of the xy-plane except a circle
   (d) The area inside a circle, including the circle
   (e) The area inside a circle, not including the circle
   (f) The area outside a circle, including the circle
   (g) The area outside a circle, not including the circle
   (h) The first quadrant
   (i) The first and third quadrants

2. Which of the following equations describes a level curve of the function $f(x, y) = e^{x^2 - y}$?

   (a) $\ln y = e^{x^2}$
   (b) $y = x^2$
   (c) $y = x$
   (d) $y = 2x$
   (e) $y = 2xe^{x^2}$
   (f) $y = e^{-x^2}$
   (g) $y = \ln(x)$
   (h) $y = \ln(x^2)$
   (i) $y = e^x$
   (j) $y = e^{x^2}$

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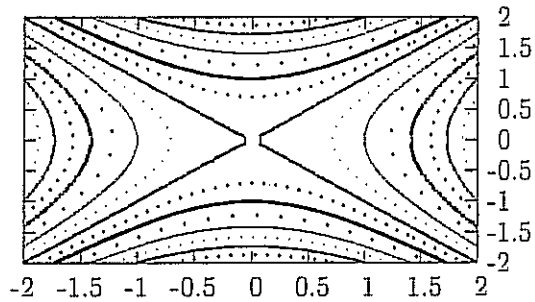
3. Which of the following functions corresponds to the contour map below?



(a) $f(x,y) = x + y$
(b) $f(x,y) = x^2 + y^2$
(c) $f(x,y) = x^2 - y^2$
(d) $f(x,y) = 1 - x^2 - y^2$
(e) $f(x,y) = xy$

4. Let $f(x,y) = \ln(x^2 y)$. What is $\frac{\partial f}{\partial x}$ at the point $(1,2)$?

(a) 0
(b) 1/8
(c) 1/4
(d) 1/2
(e) $\ln 2$
(f) 1
(g) $\ln 4$
(h) $e/2$
(i) 2
(j) e

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5. Let $f(x, y, z) = 2x^2 + 3xy + 5y^2 + z^2 - 2z$. Which of the following is a critical point of f?

   (a) $(0, 0, 0)$
   (b) $(0, 0, 1)$
   (c) $(0, 0, 3)$
   (d) $(1, -1, 0)$
   (e) $(1, -1, 1)$
   (f) $(1, 1, -1)$
   (g) $(1, 1, 1)$
   (h) $(2, 0, 2)$
   (i) $(2, 5, 1)$
   (j) $(3, 3, 2)$

6. The function $f(x, y) = x^2 + 6y^2 + 4xy - 2x + 4y$ has one critical point. What is the value of $f(x, y)$ at the critical point?

   (a) -12
   (b) -9
   (c) -3
   (d) -1
   (e) 0
   (f) 1
   (g) 3
   (h) 9
   (i) 12

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7. The point $(0,0)$ is a critical point of all the following functions. For which of the functions is $(0,0)$ a local maximum?

   I. $f(x,y) = x^2 + y^2 - xy$
   II. $f(x,y) = x^2 + y^2 - 3xy$
   III. $f(x,y) = xy - x^2 - y^2$
   IV. $f(x,y) = 3xy - x^2 - y^2$

   (a) I only
   (b) II only
   (c) III only
   (d) IV only
   (e) I and II only
   (f) III and IV only
   (g) I and III only
   (h) II and IV only
   (i) none of them
   (j) all of them

8. Which of the following best describes the function $f(x,y) = x^2y + 2x - y$ at the point $(1,-1)$?

   (a) Local maximum
   (b) Local minimum
   (c) Saddle point
   (d) Critical point, 2nd derivative test inconclusive
   (e) Not a critical point

9. Which of the following best describes the function $f(x,y) = x^3 + y^3 - 3xy$ at the point $(1,1)$?

   (a) Local maximum
   (b) Local minimum
   (c) Saddle point
   (d) Critical point, 2nd derivative test inconclusive
   (e) Not a critical point

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10. Which of the following best describes the function $f(x, y) = x^3 - 3xy^2 + 6y^2$ at the point $(1, 0)$?

    (a) Local maximum
    (b) Local minimum
    (c) Saddle point
    (d) Critical point, 2nd derivative test inconclusive
    (e) Not a critical point

11. Find the maximum value of $f(x, y) = x^2 + 2y^2 - 6x + 1$ on the region $x^2 + y^2 \leq 25$.

    (a) -9
    (b) -4
    (c) 0
    (d) 1
    (e) 10
    (f) 16
    (g) 50
    (h) 56
    (i) 60
    (j) 72

12. Suppose that the demand function for hybrid cars in the U.S. is approximated by a function $f(g, c, m)$, where g is the average price of gas, c is the average difference in cost between hybrid and conventional cars, and m is the overall average income in the U.S. Which of the partial derivatives of f will be positive?

    (a) None of them
    (b) $\frac{\partial f}{\partial g}$ only
    (c) $\frac{\partial f}{\partial c}$ only
    (d) $\frac{\partial f}{\partial m}$ only
    (e) $\frac{\partial f}{\partial g}$ and $\frac{\partial f}{\partial c}$ only
    (f) $\frac{\partial f}{\partial g}$ and $\frac{\partial f}{\partial m}$ only
    (g) $\frac{\partial f}{\partial c}$ and $\frac{\partial f}{\partial m}$ only
    (h) All three of them

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WRITTEN PROBLEM—SHOW YOUR WORK

13. (20 pts) A farmer that grows apples and oranges is able to sell apples for \$2 a pound and oranges for \$3 a pound. The cost of producing x pounds of apples and y pounds of oranges is $C(x, y) = x + y + \frac{1}{1000}(x^2 + y^2)$. Find the values of x and y that maximize profit.

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14. (20 pts) Use Lagrange multipliers to find the minimum value of $f(x, y) = x + 4y$, subject to the constraints that $xy = 1$ and (x, y) lies in the first quadrant.