Name:
ID:
Discussion Section:

This exam has 16 questions:

- 14 multiple choice worth 5 points each.
- 2 hand graded worth 15 points each.

Important:

- No graphing calculators!
- For the multiple choice questions, mark your answer on the answer card. This is the only portion of this part of the exam that will be graded.
- Show all your work for the written problems. You will be graded on the ease of reading your solution.
- You are allowed a 3×5 note card for the exam.

1. The function below has two critical points (a, b) and (c, d).

$$f(x, y) = x^3 + y^2 - 2xy + 7x - 8y + 4$$

What is the sum of the x-coordinates of the critical points, $a + c$?

(a) 0

(b) $-\frac{1}{3}$

(c) $\frac{1}{3}$

(d) $-\frac{2}{3}$

(e) $\frac{2}{3}$

(f) -1

(g) 1

(h) None of the above.

2. The function below has critical points at $(1,0)$ and $(2,1)$ (you don't have to check this).

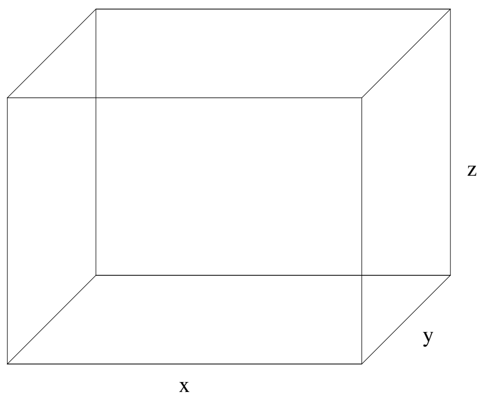
$$f(x,y) = x^3 - 3x^2 + 3x - 3xy + y^3 + 3y - 3$$

Determine the nature of these critical points.

(a) Minimum at $(1,0)$, minimum at $(2,1)$.
(b) Minimum at $(1,0)$, maximum at $(2,1)$.
(c) Minimum at $(1,0)$, saddle at $(2,1)$.
(d) Saddle at $(1,0)$, maximum at $(2,1)$.
(e) Saddle at $(1,0)$, minimum at $(2,1)$.
(f) Maximum at $(1,0)$, minimum at $(2,1)$.
(g) Maximum at $(1,0)$, saddle at $(2,1)$.
(h) Maximum at $(1,0)$, maximum at $(2,1)$.
(i) None of the above.

3. A building in the shape of a rectangular box is to have volume $12,000$ cubic feet (see the figure). It is estimated that the annual heating and cooling costs will be \$2 per square foot for the top, \$4 per square foot for the front and back, and \$3 per square foot for the sides.

   The builders want to minimize these heating/cooling costs.

   Find the cost function for these heating/cooling costs.

   (a) $C(x, y, z) = xy + yz + xz$
   (b) $C(x, y, z) = xy + 2yz + 2xz$
   (c) $C(x, y, z) = 2xy + 2yz + 2xz$
   (d) $C(x, y, z) = 2xy + 3yz + 4xz$
   (e) $C(x, y, z) = 2xy + 4yz + 3xz$
   (f) $C(x, y, z) = 2xy + 6yz + 8xz$
   (g) $C(x, y, z) = 2xy + 8yz + 6xz$
   (h) $C(x, y, z) = 4xy + 6yz + 8xz$
   (i) $C(x, y, z) = 12,000$
   (j) None of the above.

4. In Question 3, you were asked to find the cost function. There should have also been a constraint in that problem. What is the constraint?

   (a) $2xy + 4yz + 3xz = 12$
   (b) $2xy + 4yz + 3xz = 12,000$
   (c) $2xy + 6yz + 8xz = 12$
   (d) $2xy + 6yz + 8xz = 12,000$
   (e) $xy = 12$
   (f) $xy = 12,000$
   (g) $xyz = 12,000$
   (h) $xyz^2 = 12$
   (i) $xyz^2 = 12,000$
   (j) None of the above.

5. The following bits of information are known about a certain function $f(x, y)$:

   - $f(5, 13) = -43$
   - $f_x(5, 13) = 0$
   - $f_y(5, 13) = 0$
   - $f_{xx}(5, 13) = -5$
   - $f_{yy}(5, 13) = -12$
   - $f_{yx}(5, 13) = 7$

   Which of the following statements are true:

   I. $(5, 13)$ is a critical point of f.
   II. $(5, 13)$ is not critical point of f.
   III. There is a saddle at $(5, 13)$.
   IV. There is a maximum at $(5, 13)$.
   V. There is a minimum at $(5, 13)$.
   VI. There is an inflection point at $(5, 13)$.

   (a) I only
   (b) II only
   (c) VI only
   (d) I and III only
   (e) I and IV only
   (f) I and V only
   (g) I and VI only
   (h) I, II, III, IV, V, VI are all true.
   (i) I, II, III, IV, V, VI are all false
   (j) None of the above

6. Using Lagrange multipliers, as done in class,
   minimize $f(x, y) = x^2 + y^2$ subject to the constraint $x + 2y = 10$.

   Using Lagrange multipliers you should find one minimum at the point (a, b) and λ.

   What is $a + b + \lambda$?

   (a) -1
   (b) -2
   (c) 0
   (d) 1
   (e) 2
   (f) 3
   (g) 4
   (h) 5
   (i) 6
   (j) None of the above

7. In a study of five industrial areas, a researcher obtained these data relating the average number of units of a certain pollutant in the air and the incidence (per 100,000 people) of a certain disease.

Units of Pollutant	**Incidence of disease**
3.4	48
4.6	52
5.2	53
8.0	76

Let x be the number of units of pollutant and let y be the incident of disease (number of people with disease per 100,000 people).

Find the equation of the least squares line $y = ax + b$.

What is $\dfrac{b}{a}$ (round to 1 decimal place)?

(a) 1.3
(b) 2.7
(c) 3.7
(d) 4.5
(e) 11.1
(f) 13.8
(g) 26.1
(h) None of the above.

8. For each of three different years, the table below gives gives the percentage of high school students who had used cocaine at least once in their lives up to that year.

Year	**Percentage**
1995	6.0
2000	7.0
2004	9.5

Use the least squares line to predict the percentage of high school students who will use cocaine in the year 2007 (round to one decimal place).

(Hint: While not necessary, it may be useful to let t be the number of years past 1995.)

(a) 0.2
(b) 8.4
(c) 10.0
(d) 10.3
(e) 10.7
(f) 11.4
(g) 12.7
(h) 99.9
(i) None of the above

9. A manufacturer estimates that when x units of a particular commodity are sold domestically and y units are sold to foreign markets, the profit, in hundred of dollars, is given by

$$P(x, y) = (x - 30)(70 + 5x - 4y)$$

If monthly domestic sales vary between 60 and 90 units and foreign sales vary between 70 and 90 units, what is the average monthly profit?

(Hint: Watch your units!)

(a) \$3,600
(b) \$6,000
(c) \$36,000
(d) \$60,000
(e) \$360,000
(f) \$600,000
(g) \$3,600,000
(h) \$6,000,000
(i) None of the above

10. A community is laid out as a rectangular grid in relation to two main streets that intersect at the city center. Each point in the community has coordinates (x, y) in this grid, for $-5 \leq x \leq 5$, $-5 \leq y \leq 5$ with x and y measured in miles. Suppose the value of the land located at the point (x, y) is V thousand dollars per square mile where

$$V(x, y) = 300x^2 e^{-y}$$

Estimate the value (rounded to the nearest thousand dollars) of the block of land occupying the rectangular region
$1 \leq x \leq 2$, $-1 \leq y \leq 0$.

(a) \$1,000
(b) \$2,000
(c) \$815,000
(d) \$1,203,000
(e) \$1,284,000
(f) \$1,547,000
(g) \$3,262,000
(h) \$10,231,000
(i) \$3,710,161,000
(j) None of the above

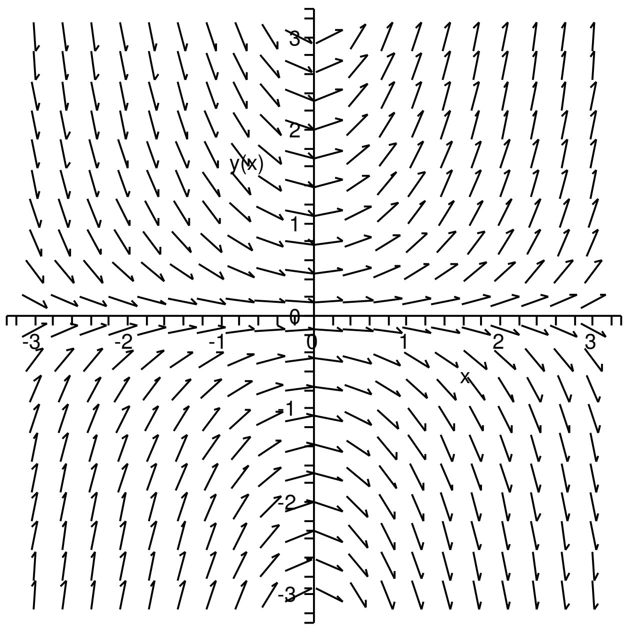
11. Consider the differential equation

$$y'' = y' + 2$$

Which of the following are a solution to the differential equation?

(a) $y = x^3 + Cx$
(b) $y = 4e^x - 2x + 7$
(c) $y = 4x + 7$
(d) $y = x^3 - 7x^2$
(e) $y = \ln x$
(f) More than one of the above is a solution.
(g) None of the above

12. Consider the slope field pictured below.



Which differential equation gives the given slope field?

(a) $y' = y$

(b) $y' = x$

(c) $y' = y/x$

(d) $y' = x/y$

(e) $y' = xy$

(f) $y' = yx^2$

(g) None of the above

13. Solve the initial value problem and find $y(-1)$.

$$y' = x^2 y^2, \qquad y(0) = 3$$

What is $y(-1)$?

(a) -3
(b) 0
(c) $\frac{1}{2}$
(d) 1
(e) $\frac{3}{2}$
(f) 2
(g) $\frac{5}{2}$
(h) 3
(i) None of the above

14. When the breath is held, carbon dioxide (CO_2) diffuses from the blood into the lungs at a steadily decreasing rate:

    - Let P_0 denote the pressure of CO_2 in the lungs at the moment when breath is held. Let P_b denote the pressure of CO_2 in the blood at the moment when breath is held. (P_0 and P_b are both constants.)
    - Assume that the pressure of CO_2 in the blood is constant, P_b, while the breath is held but the pressure in the lungs is not constant.
    - Let $P(t)$ denote the pressure of CO_2 in the lungs at time $t \geq 0$.
    - The rate of change of $P(t)$ is proportional to the difference between the two pressures $P(t)$ and P_b.

    Find The differential equation that models the diffusion of CO_2 in the lungs during breath holding.

    (a) $P'(t) = \dfrac{k}{P_0 - P_b}$

    (b) $P'(t) = \dfrac{k}{P(t) - P_0}$

    (c) $P'(t) = \dfrac{k}{P(t) - P_b}$

    (d) $P'(t) = k\,[P_0 - P_b]$

    (e) $P'(t) = k\,[P(t) - P_0]$

    (f) $P'(t) = k\,[P(t) - P_b]$

    (g) None of the above

WRITTEN PROBLEM—SHOW YOUR WORK

Name:
ID:
Discussion Section:

15. A company is developing a new soft drink. The cost, in dollars, to produce a batch of the soft drink is

$$C(x,y) = 2200 + 27x^3 - 72xy + 8y^2$$

where x is the number of kilograms of sugar per batch and y is the number of grams of flavoring per batch.

The point of this problem is to minimize the cost function.

(a) Find all critical points of $C(x,y)$.

(b) Test all critical points to determine what types of critical point they are.

(c) Find the amounts of sugar and flavoring that minimize cost.

(d) Find the minimum cost.

WRITTEN PROBLEM—SHOW YOUR WORK

Name:
ID:
Discussion Section:

16. The profit from the sale of x radiators for automobiles and y radiators for generators is given by

$$P(x, y) = -x^2 - y^2 + 4x + 8y$$

A total of 6 radiators are to be produced.

The point of this problem is to maximize the profit function using Lagrange multipliers.

(a) What is the function to maximize?

(b) There should be a constraint, what is it?

(c) Using Lagrange multipliers, find the numbers of each type of radiator to produce that maximize profit.

(d) What is this maximum profit?