Math 128
Midterm Examination 3 – November 11, 2008

Name __________________________

6 problems, 100 points.

Instructions: Show all work – partial credit will be given, and "Answers without work are worth credit without points." You don't have to simplify your answers. You may use a simple calculator that is not graphing or programmable. You may have a 3x5 card, but no other notes.

1. (16 points) Let $y' = x + y$, and suppose $y(0) = 0$. Use Euler's method with four steps ($n = 4$) to approximate $y(1)$.

2. (a) (8 points) Find the 3rd degree Taylor polynomial $T_3(x)$ centered at 0 of the function $f(x) = \ln(1+x)$.

(b) (4 points) Find the exact value of $\sum_{k=0}^{\infty} x^{2k}$ at $x = \frac{2}{3}$.

(c) (4 points) Find the exact value of $\sum_{k=0}^{\infty} x^{2k}$ at $x = \frac{3}{2}$.

3. Consider the function $f(x) = \sin x^2$.

   (a) (7 points) Find the Taylor series centered at 0 for $\sin x^2$.

   (b) (7 points) Using part (a), evaluate the integral $\int \sin x^2 \, dx$.

4. Let $f(x) = \sin 2x$.

(a) (4 points) Find the 3rd degree Taylor polynomial $T_3(x)$ centered at 0 approximating $\sin 2x$.

(b) (4 points) What is the *exact* error of your $T_3(x)$ from part (a) at $x = \dfrac{\pi}{4}$?

(c) (12 points) Let $T_n(x)$ be the degree n Taylor polynomial for $\sin 2x$, centered at 0. Find an upper bound for the error of $T_n(x)$ on the interval $[-1, 1]$. Your bound should depend on n.

5. (15 points) Let $f(x) = x \sin x^2$. Calculate $f^{(14)}(0)$ and $f^{(15)}(0)$.
Hint: Use the coefficients of the Taylor series.

6. For each of the following series, show why it converges or diverges:

(a) (7 points) $\displaystyle\sum_{k=1}^{\infty} \frac{\sin^2 k}{k^{5/4}}$.

(b) (7 points) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$.

You may use the fact that $\dfrac{\ln x}{x^{3/2}}$ is decreasing on the interval $[2, \infty)$.

(c) (5 points) $\displaystyle\sum_{j=1}^{\infty} \frac{j+1}{j^2+1}$.