Math 128 Final Exam
December 17, 2003

This exam has 23 questions.

For answers in dollars, round to the nearest dollar

1. You have an income stream of $1000e^{-.07t}$. As the money comes in you invest it at an interest rate of 8%, compounded continuously. How much money will there be in 10 years?

   (a) $\$11,458$
   (b) $\$11,526$
   (c) $\$11,578$
   (d) $\$11,593$
   (e) $\$11,607$
   (f) $\$11,632$
   (g) $\$11,665$
   (h) $\$11,687$

2. How much of the money in the previous answer is interest?

   (a) $\$4,135$
   (b) $\$4,435$
   (c) $\$4,546$
   (d) $\$4,646$
   (e) $\$4,715$
   (f) $\$4,735$
   (g) $\$4,815$
   (h) $\$4,888$

3. Evaluate the integral

$$\int\int_R (x+2y)^2 \, dA$$

Where R is the rectangle $R = \{(x,y) \mid 1 < x < 2,\ 0 < y < 1\}$.

(a) 1

(b) 2

(c) 3/2

(d) 5/2

(e) 7/2

(f) 10/3

(g) 17/3

(h) 20/3

4. Find the minimum value of the function $f(x,y) = 2x^2 + 4xy - 12x + 4y^2 - 20y + 31$.

(a) 3

(b) 4

(c) 5

(d) 6

(e) 7

(f) 8

(g) 9

(h) There is no minimum

5. Find the particular solution of the differential equation $xyy' = x^2 + 1$ which has $y(e) = e$.

(a) $\sqrt{x^2 + x^4 - e^4}$

(b) $x + (x - e)^2$

(c) $x + \sqrt{2\ln x - 2}$

(d) $x + (x - e)^3$

(e) $\sqrt{x^2 + (x - e)^3}$

(f) $x + \sqrt{(x - e)^3}$

(g) $x^3 - ex^2 + e$

(h) $\sqrt{x^2 + 2\ln x - 2}$

6. Suppose the US population is now $P(0) = 285$ million and that its growing in a way described by the differential equation $P'(t) = .035P(t)$. How many years will it take for the population to reach 310 million?

(a) 1.9

(b) 2.1

(c) 2.2

(d) 2.3

(e) 2.4

(f) 2.5

(g) 2.7

(h) 2.8

7. Find

$$\int \sin x \cos^2 x \ dx$$

(a) $\frac{1}{2}\sin^2 x + C$

(b) $\frac{1}{3}\sin^3 x + C$

(c) $2\sin^2 x \cos^2 x + C$

(d) $\frac{-1}{3}\cos^3 x + C$

(e) $2\cos^2 x + C$

(f) $\frac{1}{3}\sin x \cos^3 x + C$

(g) $2\sin x \cos x + C$

(h) $\frac{1}{2}\cos^2 x + C$

8. Evaluate

$$\int_0^1 x \cos x \ dx$$

(a) $\cos 1 - \sin 1 - 1$

(b) $\cos 1 + \sin 1 + 1$

(c) $-\cos 1 + \sin 1 - 1$

(d) $-\cos 1 - \sin 1 - 1$

(e) $\cos 1 + \sin 1 - 1$

(f) $\cos 1 - \sin 1$

(g) $-\cos 1 - 1$

(h) $\cos 1 - \sin 1$

9. Consider the probability density function

$$f(x) = \begin{cases} \frac{3}{2}(1 - x^2) & \text{if} \quad 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the mean and the variance.

(a) mean $= \frac{1}{8}$, variance $= \frac{19}{320}$

(b) mean $= \frac{1}{4}$, variance $= \frac{19}{320}$

(c) mean $= \frac{3}{8}$, variance $= \frac{19}{320}$

(d) mean $= \frac{5}{8}$, variance $= \frac{19}{300}$

(e) mean $= \frac{7}{8}$, variance $= \frac{19}{300}$

(f) mean $= \frac{3}{4}$, variance $= \frac{19}{300}$

(g) mean $= \frac{1}{5}$, variance $= \frac{19}{360}$

(h) mean $= \frac{1}{3}$, variance $= \frac{19}{360}$

10. The lifetime of a light bulb is described by an exponential random variable with mean 100 hours. What is the probability that the bulb will last more than 100 hours?

(a) $1/e$

(b) $2/e$

(c) $1/e^2$

(d) $2/e^2$

(e) $3/e^2$

(f) $e/(100 + e)$

(g) $(100 + e)/(100e)$

(h) $e/100$

11. Phone calls come in to a switchboard at a rate of 1 per minute. What is the probability that two or more arrive in the next minute?

(a) .316
(b) .284
(c) .264
(d) .246
(e) .238
(f) .225
(g) .219
(h) .211

12. What are the first few terms of the Taylor series of $f(x) = 2xe^{x^3}$ at $a = 0$?

(a)
(b) $2x + \frac{2}{3}x^4 + \frac{1}{24}x^7 + \cdots$.
(c) $2 + 2x + 2x^4 + 2x^7 + \cdots$.
(d) $2 + 2x^4 + 2x^7 + \cdots$.
(e) $2x + 2x^4 + x^7 + \cdots$.
(f) $2x + \frac{1}{4}x^4 + \frac{1}{7}x^7 + \cdots$.
(g) $2x + \frac{2}{4!}x^4 + \frac{1}{7!}x^7 + \cdots$.
(h) $x + +\frac{2}{4!}x^4 + \frac{1}{7!}x^7 + \cdots$.
(i) $x^3 + \frac{1}{2}x^6 + \frac{1}{3}x^9 + \cdots$.

13. The weight of the Cheerios in a "15 oz." box is normally distributed with mean 15.1 and standard deviation .2. Compute a and b so that the probability that a box will contain between 14.8 and 15.2 oz. of cereal is

$$\frac{1}{\sqrt{2\pi}}\int_a^b e^{-x^2/2}dx.$$

(a)
(b) $a = -\infty$, $b = -.3$
(c) $a = -\infty$, $b = 14.8$
(d) $a = -\infty$, $b = 15.2$
(e) $a = -\infty$, $b = .5$
(f) $a = -1.5$, $b = .5$
(g) $a = -\infty$, $b = -1.5$
(h) $a = -1.5$, $b = .5$
(i) $a = -1.5$, $b = \infty$

14. What value do you get when you approximate $\cos(.1)$ using $p_2(x)$, the second degree Taylor polynomial of $\cos(x)$ at the base point $a = 0$?

(a)
(b) .995000
(c) .995001
(d) .995002
(e) .995003
(f) .995004
(g) .995005
(h) .995006
(i) .995007

15. For $g(x) = \sin 2x + \cos^2 x$ find $g'\left(\frac{\pi}{4}\right)$.

(a) -2

(b) -1

(c) -0

(d) 1

(e) 2

(f) $1/2$

(g) $-1/2$

(h) $\sqrt{2}$

16. Find the area between the graphs of $y = \sin x$ and $y = \cos x$ for $0 < x < \pi/2$.

(a) 0

(b) $\sqrt{2}/2 - 1$

(c) $\sqrt{2}/2 + 1$

(d) $\sqrt{2}/2 - 2$

(e) $\sqrt{2}/2 + 2$

(f) $2\sqrt{2}$

(g) $2\sqrt{2} + 2$

(h) $2\sqrt{2} - 2$

17. Evaluate the improper integral

$$\int_1^\infty \frac{3}{x^{3/2}} dx$$

(a) 0

(b) 1

(c) 2

(d) 3

(e) 4

(f) 5

(g) 6 ∗ ∗

(h) The integral diverges

18. Find the interval of convergence of the Taylor series

$$\frac{1}{(3+2x)^2} = \frac{1}{9} - \frac{4}{27}x + \frac{4}{27}x^2 - \frac{32}{243}x^3 \cdots + (-1)^n (n+1) \frac{2^n}{3^{n+2}} x^n + \cdots$$

(a) $x = 0$ only.

(b) $(-.5, .5)$

(c) $(-1, 1)$

(d) $(-1.5, 1.5)$

(e) $(0, 2)$

(f) $(-2/3, 2/3)$

(g) $(0, 4/3)$

(h) The series converges for all x.

19. True or False:

    If money is invested at an interest rate of $r\%$,compounded continuously, then the money will double in approximately $70/r$ years.

    (a) True

    (b) False

20. True or False:

    For every value of x, Taylor series for the function $\frac{1}{1-x}$ converges to $\frac{1}{1-x}$.

    (a) True

    (b) False

21. True or False:

    If a continuous random variable has median M then the probability that the quantity is less than M equals the probability that the quantity is greater than M.

    (a) True

    (b) False

22. True or False:

    One can find the minimum of the function $x^2 + 5y^2 + 6x + 9$ using the method of Lagrange multipliers.

    (a) True

    (b) False

23. True or False:

If $f(x)$ is a probability density function then for all x, $0 \leq f(x) \leq 1$.

(a) True

(b) False