This exam has 23 questions. For questions 1 through 21 indicate your answer on the answer card. Problems 22 and 23 will be graded by hand. For those two questions you must indicate work which justifies your answer to receive full credit.

For answers in dollars, round to the nearest dollar

1. Find the area between the graph of \( y = x^3 - 3x \) and the graph of \( y = x \)?

   a. 2
   b. 4
   c. 6
   d. 8
   e. 16
   f. 7/2
   g. 5/2
   h. 3/2
   i. 1/2
   j. 0
2. \( f(x,y) = xy^2 + y + (1 + 2xy)^2 \). Find \( f_{yy}(2,1) \).
   
   a. 144
   b. 128
   c. 108
   d. 96
   e. 84
   f. 64
   g. 56
   h. 48
   i. 36
   j. 18

3. The number of local maxima, local minima, and saddle points of the function \( f(x,y) = x^3 + y^3 - xy \) are A, B, and C respectively. \((A,B,C) = ?\)
   
   a. \((1,0,1)\)
   b. \((0,2,0)\)
   c. \((1,1,1)\)
   d. \((1,1,0)\)
   e. \((2,0,1)\)
   f. \((0,1,1)\)
   g. \((0,2,1)\)
   h. \((0,0,0)\)
   i. \((1,0,2)\)
   j. \((2,1,1)\)
4. Find the antiderivative

\[ \int (x + 3x^2y^2) \, dx. \]

a. \( x + 3x^2y^2/2 + c(y) \)
b. \( x + 3x^2y^2/2 + c(x) \)
c. \( 1 + 6xy + c(x) \)
d. \( 3x^2 + c(y) \)
e. \( x^2/2 + 3xy^3 + c(y) \)
f. \( x^2y^3 + c(x) \)
g. \( x^2/2 + x^3y + c(x) \)
h. \( x^2/2 + x^3y + c(y) \)
i. \( x^2 + 3xy^3 + c(y) \)
j. \( x^2/2 + x^2y^3 + c(y) \)

5. Use the linear regression line for the data

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>4</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>3</td>
<td>5</td>
<td>9</td>
<td>15</td>
</tr>
</tbody>
</table>

to predict the \( y \) value corresponding to \( x = 7 \)?

a. 7.2
b. 7.3
c. 7.4
d. 7.5
e. 7.6
f. 7.7
g. 7.8
h. 7.9
i. 8.0
j. 8.1
6. Find the minimum, if there is one, of the function $f(x, y) = 2x^2 - 6y - 16x + 3y^2 + 40$.

   a. 7
   b. 5
   c. 3
   d. 1
   e. -1
   f. 0
   g. 2
   h. 4
   i. 6
   j. There is no minimum.

7. Find the minimum, if there is one, of the function $4x^2 - y^2$.

   a. -24
   b. -18
   c. -16
   d. -9
   e. 0
   f. 15
   g. 32
   h. 48
   i. 64
   j. There is no minimum.
8. Suppose you have an income stream that produces income of $f(t) = 1000e^{0.1t}$ at time $t$ and that as you receive the income you invest it at an interest rate of 5%. What will be the accumulated value (the future value) in 10 years?

a. $22,311  
   b. $22,411  
   c. $22,511  
   d. $22,611  
   e. $22,711  
   f. $22,811  
   g. $22,911  
   h. $23,011  
   i. $23,111  
   j. $23,211

9. What is the volume under the surface $z = \frac{y}{x}$ above the rectangle \( \{(x,y) : 1 \leq x \leq 2, 0 \leq y \leq 2\} \).

a. 0  
b. 1  
c. 2 \ln 2  
d. 3 \ln 3  
e. 4 \ln 2  
f. 1/2  
g. 3/2  
h. 5/2  
i. 7/2  
j. 9/2
10. You are planning to invest $120,000 in a production facility whose output is described by the Cobb-Douglas production function \( N(x,y) = 10x^{2/3}y^{1/3} \). Here \( N(x,y) \) is the number of units of output produced by \( x \) units of labor and \( y \) units of capital. If a unit of labor costs $20 and a unit of capital costs $40, what is the maximum output that can be achieved?

   a. 20,123  
   b. 20,735  
   c. 20,938  
   d. 21,237  
   e. 22,543  
   f. 23,869  
   g. 24,213  
   h. 25,198  
   i. 27,884  
   j. 29,016  
   k. 31,349

11. What is the marginal productivity of money in the situation described in the previous problem?

   a. .19542  
   b. .19894  
   c. .20999  
   d. .21376  
   e. .21897  
   f. .22623  
   g. .22999  
   h. .23122  
   i. .23589  
   j. .24120
12. For the function \( f(x, y) = e^{2x}y^2 \), find (and simplify)

\[
\frac{f(x, y + k) - f(x, y)}{k}
\]

a. \( e^{2(x+k)}(y - k) \)
b. \( e^{2x}(2y + k)/k \)
c. \( e^{2k}(y + k)^2 \)
d. \( e^{2x}(y + k)^2 - e^{2x}y^2 \)
e. \( e^{2x}k \)
f. \( e^{2(x+k)}(y + k)^2 - e^{2x}y^2 \)
g. \( e^{2x}(2y + k) \)
h. \( (2y + k) \)
i. \( e^{2x}(2y - k) \)
j. \( e^{2x}(-2y + k) \)

13. The general solution of the differential equation \( y' = 4(100 - y) \) is \( y = 100 - ce^{4x} \). Find the particular solution which satisfies the initial condition \( y(0) = 40 \).

a. \( 4(100 - 0) - 360 \)
b. \( 100 - 60e^{4x} \)
c. \( 40 - ce^{4x} \)
d. \( 41 - e^{4x} \)
e. \( 40e^{4x} \)
f. \( 100 - 40e^{4x} \)
g. \( 140 - 100e^{4x} \)
h. \( 60 + 40e^{4x} \)
i. \( 100 - 40e^{x} \)
j. \( 60 - 40e^{4x} \)
14. Evaluate:

\[ \int_0^2 x^2 e^x \, dx \]

a. \( \frac{1}{3} \)

b. \( \frac{2}{3} \)

c. \( 1 \)

d. \( e^2 - e \)

e. \( -2e^2 + 3e \)

f. \( 4e - 2e^2 - 1 \)

g. \( e^2 - 2e \)

h. \( 6e + e^2 \)

i. \( 4e^2 - 2e^e + 2 \)

j. \( 2e^2 - 2 \)

15. Evaluate:

\[ \int_1^2 x^{-2} \ln x \, dx \]

a. \( 3 - 2\ln 2 \)

b. \( (1 + \ln 2)/4 \)

c. \( 4\ln 2 \)

d. \( (1 - \ln 2)/2 \)

e. \( \ln 2 - 1 \)

f. \( 3\ln 2 - 2 \)

g. \( 2\ln 2 + 1 \)

h. \( 2\ln 2 + 2 \)

i. \( (2 + \ln 2)/4 \)

j. \( (2 - \ln 2)/4 \)
TRUE OR FALSE

16. The future value of an income stream is always greater than the present value.
   a. True
   b. False

17. If you have the regression line for some data points \((x_1, y_1), \ldots, (x_n, y_n)\) then the slope of that line is a measure of the rate of change of \(y\) as \(x\) varies.
   a. True
   b. False

18. The function \(y\) defined implicitly by \(y + e^y = x^2\) is a solution of the differential equation \((x^2 + 1 - y)y' = 2x\) which satisfies the initial condition \(y(1) = 0\).
   a. True
   b. False

19. If \(K(x, y)\) is the output of a factory given input of \(x\) units of labor and \(y\) units of capital then the second partial derivative, \(K_{xx}(x, y)\), gives the marginal productivity of labor.
   a. True
   b. False

20. Saying that the marginal productivity of money for a factory is .021 means that each additional dollar spent on production would result in an increase in production of roughly .021 units.
   a. True
   b. False

21. The regression line is a line that goes through all the data points.
   a. True
   b. False
This question will be hand graded, to get full credit in needs to be clear HOW you arrived at your answer.

22. Use the method of Lagrange multipliers to find the maximum of the function \( f(x, y) = 2xy \) subject to the condition \( 2x + 4y = 24 \).
This question will be hand graded, to get full credit in needs to be clear HOW you arrived at your answer.

23. Evaluate

\[ \int \int_R xe^{2y} \, dA \]

where \( R \) is the rectangle \( R = \{(x,y) : 0 \leq x \leq 1, 1 \leq y \leq 2\} \).