Math 128

Practice Final

December 2003

This exam has 23 questions.

For answers in dollars, round to the nearest dollar

1. You win $1 million in the lottery and will be paid $50,000 per year for 20 years. Assuming an interest rate of 8%, what is the present value of that payout?

   a. $726,815
   b. $715,815
   c. $676,815
   d. $643,815
   e. $603,815
   f. $569,815
   g. $513,815
   h. $498,815

2. You have an income stream of $1000e^{0.02t}. As the money comes in you invest it at an interest rate of 8%, compounded continuously. How much interest will you accumulate in 10 years?

   a. $3,025
   b. $3,147
   c. $3,756
   d. $4,335
   e. $4,872
   f. $5,022
   g. $5,148
   h. $5,666
3. What is the average value of the function \( f(x, y) = \sin(x + y) \) on the square \( R = \{(x, y) | 0 < x < \pi/4, 0 < y < \pi/4\} \)?

   a. \( \frac{1}{\pi^2} \left( \sqrt{2} - 1 \right) \)
   b. \( \frac{4}{\pi^2} \left( \sqrt{2} - 1 \right) \)
   c. \( \frac{16}{\pi^2} \left( \sqrt{2} - 1 \right) \)
   d. \( \frac{1}{\pi^2} \left( \sqrt{2} - 1 \right) \)
   e. \( (2 - \sqrt{2}) \)
   f. \( \frac{1}{4} \left( \sqrt{3} - \sqrt{2} \right) \)
   g. \( \frac{1}{2} \left( 2\sqrt{2} - 1 \right) \)
   h. \( 4\sqrt{2} - 2 \)

4. Minimize \( x^2 + 4y^2 + 5 \) subject to the constraint \( xy = 1 \).

   a. There is no minimum.
   b. 4
   c. 5
   d. 6
   e. 7
   f. 8
   g. 9
   h. 10
5. The Lorenz curve for income distribution in a country is given by the formula \( f(x) = e^{-1}xe^x \). Compute the associated Gini index.

   a. \( 1/(e - 2) \)
   b. \( 1 - 2/e \)
   c. \( 1 - 1/e \)
   d. \( 1/2 + 1/e \)
   e. \( 1/e \)
   f. \( e/3 \)
   g. \( e/4 \)
   h. \( 2/e \)

6. Suppose \( y(x) \) is the solution of the differential equation

\[
y' + 2xy = e^{-x^2}
\]

which satisfies the initial condition \( y(0) = 1 \). What is \( y(1) \)?

   a. \( e^2 \)
   b. \( 2e^2 \)
   c. \( e + e^2 \)
   d. \( e - e^2 \)
   e. \( e^{-1} \)
   f. \( e + e^{-1} \)
   g. \( 2e^{-1} \)
   h. \( e^2 + 1 \)
7. Newton’s law of cooling states that a hot object cools at a rate proportional to the difference between its temperature and the temperature of the surroundings. Suppose you put out a cup of coffee at 170°. The room temperature is 70°. After ten minutes the coffee has cooled to 140°. What will be the temperature of the coffee 20 minutes after you put it out?

a. 111°
b. 113°
c. 115°
d. 117°
e. 119°
f. 121°
g. 123°
h. 125°

8. \[
\int x \sin(x^2) \, dx
\]

a. \(\cos(x) \sin(x) + C\)
b. \(2x \cos(x^2) + C\)
c. \(2 \cos(x^2) + C\)
d. \(-\frac{1}{2} \sin(x^2) + C\)
e. \(2 \sin(x^2) + C\)
f. \(-\frac{1}{2} \cos(x^2) + C\)
g. \(\sin(x^2) \cos(x^2) + C\)
h. \(\cos(x^2) + 2x^2 \cos(x^2) + C\)
9. You have a coin which comes up heads with probability 1/4. What is the probability that if you toss the coin three times it will come up heads exactly one time?

   a. 27/64  
   b. 25/64  
   c. 23/64  
   d. 21/64  
   e. 13/32  
   f. 11/32  
   g. 7/16  
   h. 5/16  

10. A random variable has the density function

\[ f(x) = \begin{cases} 
3x^2 & \text{if } 0 \leq x \leq 1 \\
0 & \text{otherwise}
\end{cases} \]

What is its median?

   a. \((.5)^{1/3}\)  
   b. \((.6)^{1/3}\)  
   c. \((.7)^{1/3}\)  
   d. \((.5)^{1/2}\)  
   e. \((.3)^{1/4}\)  
   f. .76  
   g. .75  
   h. .74
11. The time between incoming phone calls is an exponential random variable with mean 2 minutes. What is the probability that the time between the next call and the one after that is between 2 and 4 minutes?

a. $e^{-1}$
b. $e^{-2}$
c. $e^{-1} - e^{-2}$
d. $e^{-1} + e^{-2}$
e. $1 - e^{-1}$
f. $1 - e^{-2}$
g. 0
h. 1

12. Use the Taylor polynomial of degree two for the function $f(x) = e^{-x^2}$ to estimate the average value of $f(x)$ on the interval $(0, 1/2)$.

a. .9167
b. .9172
c. .9178
d. .9184
e. .9189
f. .9193
g. .9198
h. .9208
13. Phone calls come in to a switchboard at a rate of 2 per minute. What is the probability that exactly 3 will arrive in the next minute?

a. \(.12\)
b. \(.13\)
c. \(.14\)
d. \(.15\)
e. \(.16\)
f. \(.17\)
g. \(.18\)
h. \(.19\)

14. What are the first few terms of the Taylor series of \(f(x) = 1 + x^2 e^{2x}\)?

a. \(1 + 2x^2 + 3x^3 + 4x^4 + \cdots\)
b. \(1 + \frac{1}{2!}x^2 + \frac{2}{3!}x^3 + \frac{2}{4!}x^4 + \cdots\)
c. \(x + 2x^3 + x^5 + 2x^7 + \cdots\)
d. \(1 + x^2 + x^4 + x^5 + \cdots\)
e. \(1 + x^2 + 2x^3 + 2x^4 + 3x^4 + \cdots\)
f. \(1 + 2x^3 + 3x^7 + 4x^7 + \cdots\)
g. \(x^2 + 2x^4 + 3x^6 + 4x^8 + \cdots\)
h. \(1 + x^2 + 2x^3 + 2x^4 + \cdots\)
15. If the average temperature in July in St. Louis is 79.3° with a standard deviation of .3°. What is the probability that the average temperature next July will be between 80° and 79° on the 4th of July in St. Louis

a. .75
b. .77
c. .79
d. .81
e. .83
f. .85
g. .87
h. .89

16. Evaluate the improper integral

\[ \int_{1}^{\infty} \frac{1}{(2x + 1)^3} \, dx \]

a. 

b. 2
c. -1
d. -2
e. 1/8
f. 1/24
g. 1/36
h. 1/48
i. The integral diverges.
17. Given the data

\[
\begin{array}{cccc}
\text{x} & 1 & 2 & 3 & 4 \\
\text{y} & 1 & 3 & 2 & 6
\end{array}
\]

use the regression line to predict the value of \( y \) that will correspond to \( x = 6 \).

a. 7.5 

b. 7.6 

c. 7.7 

d. 7.8 

e. 7.9 

f. 8.0 

g. 8.1 

h. 8.2 

18. The density function for a random variable is

\[
f(x) = \begin{cases} 
\frac{1}{2} \sin x & \text{if } 0 \leq x \leq \pi \\ 
0 & \text{otherwise}
\end{cases}
\]

What is the probability that the quantity is less than \( \pi/4 \)?

a. 0 

b. \( \frac{1}{2} \) 

c. 1 

d. \( \frac{1}{4} \sqrt{2} \) 

e. \( \frac{1}{2} \sqrt{2} \) 

f. \( \frac{1}{2} + \frac{1}{4} \sqrt{2} \) 

g. \( \frac{1}{2} - \frac{1}{4} \sqrt{2} \) 

h. \( \frac{1}{4} + \frac{1}{4} \sqrt{2} \)
19. True or False:

Lorenz curve for Country A

Lorenz curve for Country B

The Lorenz curves above are for income distribution in two countries. Country A has the more equal distribution of income.

a. True

b. False

20. For values of $x$ near 0, the function $f(x) = \ln(1 - x)$ is equal to a convergent power series.

a. True

b. False

21. True or False:

In continuously compounded interest, the amount of money grows at a rate proportional to the amount of money.

a. True

b. False
22. The graph above might be the graph of a probability density function.
   a. True
   b. False

23. True or False:
    The function $y(x) = \sin 2x$ is a solution of the differential equation $y'' + 4y = 0$.
   a. True
   b. False