Math 131 , Spring 2004
Exam 1

Name: _____ **ID#** _____

Only scientific calculators are allowed. Be sure your calculator is set for "radians", not "degrees", if you do any calculus computations with trig functions. For Parts I and II, please mark your answer on the answer card. For Part III, please solve the problems in the space provided.

Part I, Multiple Choice, 5 points/problem:

1. Suppose $f(x) = x^2$, $g(x) = \sqrt{1 + \ln x}$, and $h(x) = e^{4x}$. What is the value of $(f \circ g \circ h)(1)$?

A) 0 B) 1 C) 2 D) e^4 E) $\sqrt{5}$

F) $\sqrt{6}$ G) $2 + e^4$ H) $1 + e$ I) 5 J) 6

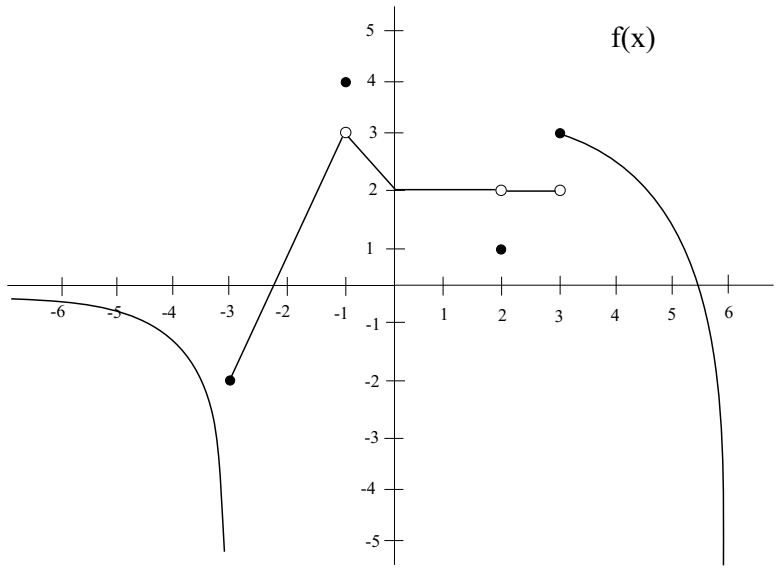
2. Find the vertical asymptotes of the function $f(x) = \dfrac{x^2 - 3x - 4}{2x^2 - 2x - 24}$.

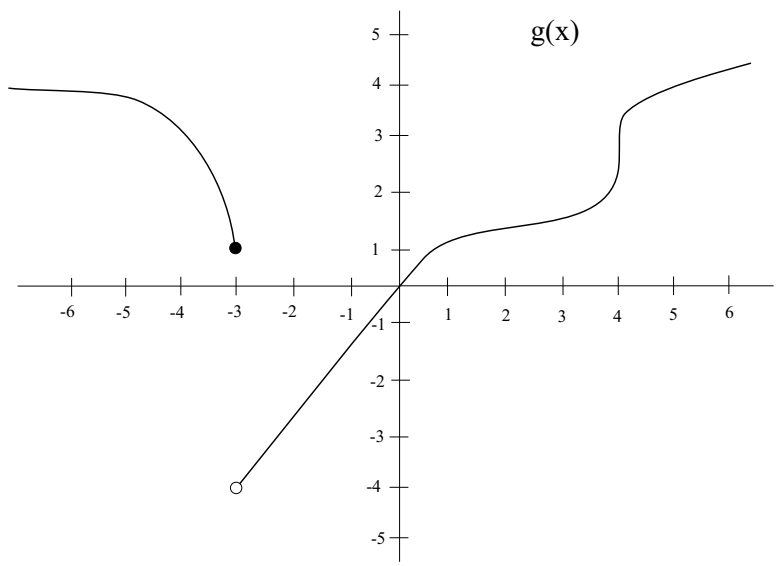
A) $x = -1$ and $x = 4$ B) $x = -3$ C) $x = \frac{1}{2}$ D) $x = -3$ and $x = 4$

E) $x = \frac{1}{3}$ and $x = 4$ F) $x = 4$ G) $x = 3$ H) $x = \frac{1}{3}$

I) $x = -1$ and $x = -3$ J) no vertical asymptotes

For questions 3 - 12, please refer to the graphs of $f(x)$ and $g(x)$ given below. Questions 3 - 7 (worth 2 points each) are "fill in the blank", where you will select your answers from those listed below the graphs. Questions 8 - 12 (worth 1 point each) are true/false questions.





The "fill in the blank" answer choices are:

A) -4 B) -3 C) -2 D) -1 E) 0

F) 1 G) 2 H) 3 I) 4 J) ∞

3. $\lim_{x \to 0} \frac{f(x)}{g(x)} =$ _____.

4. $\lim_{x \to -\infty} g(x) =$ _____.

5. $\lim_{x \to -3^-} g(x) =$ _____.

6. $\lim_{x \to 0} f(x)g(x) =$ _____.

7. $\lim_{x \to -1} f(x) =$ _____.

The following 5 questions are true/false and still refer to the functions $f(x)$ and $g(x)$, whose graphs are shown on the previous page.

8. f is discontinuous at exactly five points.

A) True B) False

9. $\lim_{x \to 2} f(x)$ does not exist.

A) True B) False

10. g is not differentiable at $x = 4$.

A) True B) False

11. f is continuous from the right at $x = 3$.

A) True B) False

12. $\lim_{x \to -3} g(x) = 1$.

A) True B) False

13. Solve the following equation for x: $\dfrac{10}{e^{-5x}+2} = 1$.

A) $x = -5\dfrac{\ln 2}{8}$ B) $x = \dfrac{8\ln 5}{2}$ C) $x = \dfrac{1}{5}$ D) $x = -10\dfrac{\ln 2}{5}$ E) $x = e^{-\frac{1}{5}}$

F) $x = \dfrac{\ln 5}{10}$ G) $x = -\dfrac{\ln 8}{5}$ H) $x = -10\dfrac{\ln 8}{2}$ I) $x = \dfrac{\ln(-2)}{10}$ J) $x = 0$

14. Suppose

$$1 + \ln(x+1) \leq f(x) \leq 2x + e^x.$$

Evaluate $\lim_{x\to 0} f(x)$.

A) $1 + \ln 2$ B) $2 + e$ C) 0 D) 2 E) $\ln 2$
F) e G) $e + \ln 2$ H) 1 I) 3 J) does not exist

15. Let $f(x) = x^2 + x + a$, where a is a constant. For which value of a will the Intermediate Value Theorem guarantee that the equation $f(x) = 0$ has a root in the interval $(0, 2)$.

A) -20 B) -8 C) -2 D) 1 E) 2
F) 4 G) 6 H) 8 I) 10 J) 12

16. What is the value of $\left(\log_2 2^6\right)\left(\ln\sqrt{e^5}\right)\left(\log_2 \frac{1}{2}\right)$?

A) -30 B) $\frac{5}{2}$ C) $\frac{1}{6}$ D) -15 E) 24

F) $-\frac{12}{5}$ G) 6 H) $-\frac{5}{4}$ I) $-\frac{e}{5}$ J) 0

17. Evaluate $\lim_{x \to 0^-} \frac{4x + |x|}{x}$.

A) ∞ B) -4 C) -3 D) -1 E) 4

F) 2 G) 0 H) 5 I) 3 J) 1

18. The table below shows the average age of marriage for women $A(t)$ in the last half of the 20th century.

t	1950	1960	1970	1980	1990
A(t)	23.0	25.0	24.2	26.5	28.6

What was the average rate of change of marriage age over the time interval $[1960, 1980]$?

A) $-\frac{6}{50}$ years/year B) $-\frac{22}{50}$ years/year C) -1.2 years/year D) $\frac{23}{100}$ years/year

E) $\frac{11}{100}$ years/year F) $\frac{3}{40}$ years/year G) $\frac{1}{50}$ years/year H) $\frac{14}{25}$ years/year

I) $\frac{3}{25}$ years/year J) 1.5 years/year

Part II: True/False (2 points each)

19. The graph of a function $f(x)$ can intersect a horizontal asymptote.

A) True B) False

20. If $\lim_{x\to 0} f(x) = 0$ and $\lim_{x\to 0} g(x) = 0$, then $\lim_{x\to 0} \frac{f(x)}{g(x)} = 1$.

A) True B) False

21. $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

A) True B) False

22. If $f(x)$ is differentiable at $x = a$, then $\lim_{x\to a} f(x) = f(a)$.

A) True B) False

23. $\lim_{x\to 0^+} \ln x = 1$

A) True B) False

Part III: These are the "free response" problems worth a total of 30 points. Write your answers on the test pages. <u>Show your work</u> neatly and cross out irrelevant scratch-work, false starts, etc.

Please put your NAME on each of the following pages, since they may be separated during grading. Also, please add your Discussion Section Letter (available on your exam front cover) on each page so that we can return these pages in your discussion section.

Name: _____ **Discussion Section:** _____

24. The $\lim_{x \to 1} \dfrac{\sqrt{x+2} - \sqrt{3}}{x-1}$ represents the derivative of some function $f(x)$ at some point a.

a) (2 points) What are f and a?

b) (4 points) Find the exact value of $f'(a)$ by evaluating the limit.

c) (2 points) Find the equation of the tangent line to $f(x)$ at the point $x = a$ (using the value for a you found in part (a)).

Name: _____ **Discussion Section:** _____

25. (12 points) Solve the following problems.

a) Evaluate $\lim_{x \to 2} \dfrac{2x^3 - 5x^2 + 2x}{x^2 + 2x - 8}$.

b) Suppose

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x \geq 1 \\ c(x-1) & \text{if } x < 1. \end{cases}$$

For what value of c will f have a derivative at $x = 1$ (i.e. will $f'(1)$ exist)?

(OVER)

c) Find the horizontal asymptotes of the function $f(x) = \sin\left(\dfrac{4x^4 - 2x + 1}{x^6 + 2x^3 - 2}\right)$.

Name: _____ **Discussion Section:** _____

26. (10 points) The following page gives graphs of five functions (labeled $f_1(x) - f_5(x)$) and graphs of their derivatives (labeled (i) - (v)). Please match the correct function with its derivative, and please explain your choice.

a) $f_1(x)$ _____ because . . .

b) $f_2(x)$ _____ because . . .

c) $f_3(x)$ _____ because . . .

d) $f_4(x)$ _____ because . . .

e) $f_5(x)$ _____ because . . .

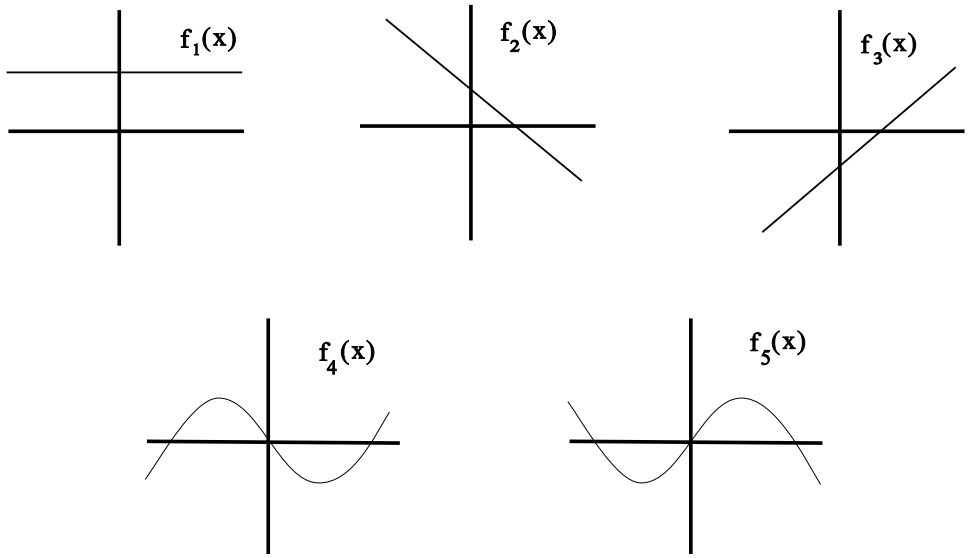


Figure 1: $f_1(x) - f_5(x)$ are functions.

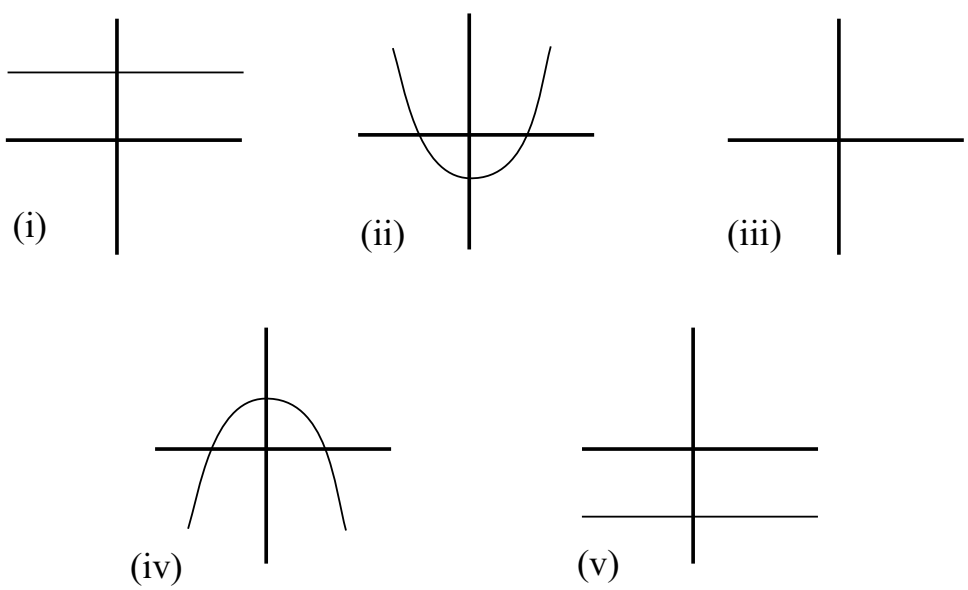


Figure 2: Graphs (i) - (v) are graphs of the derivatives of the functions $f_1(x) - f_5(x)$.