Math 131 , Spring 2004
Final Exam

Name: _____ **ID#** _____

No graphing calculators or calculators with a CAS are allowed. You may use a scientific calculator. Be sure your calculator is set for "radians", not "degrees", if you do any calculus computations with trig functions. For Part I, please mark your answer on the answer card. For Part II, please solve the problems in the space provided.

Part I, Multiple Choice, 4 points/problem:

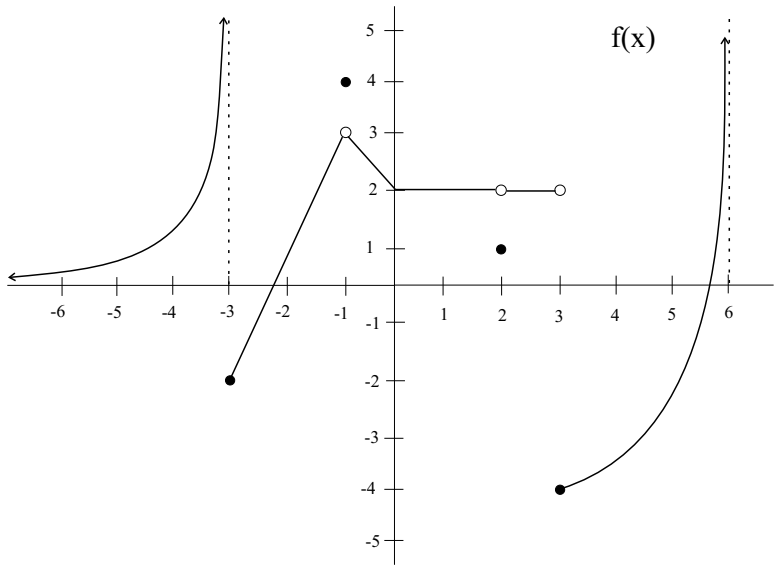
1. Which of the following is $f'(x)$ if $f(x) = \dfrac{\arctan x}{1+x^2}$. (Note that $\arctan x$ is the same as $\tan^{-1}(x)$.)

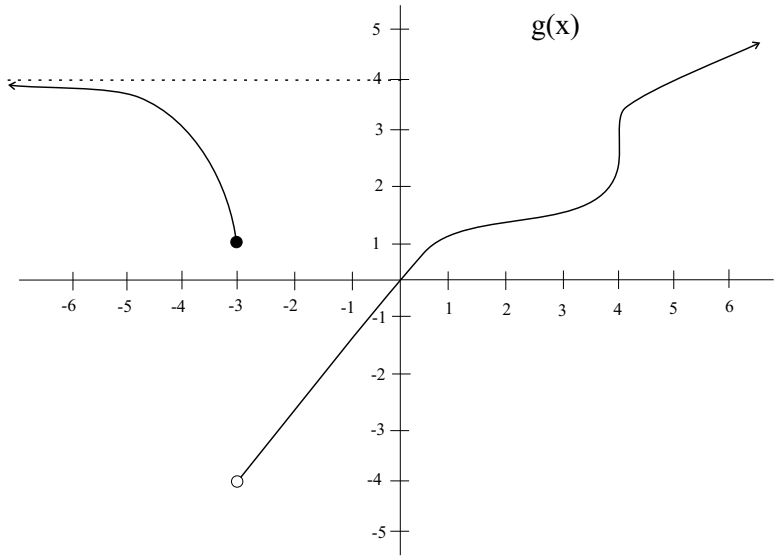
A) $\dfrac{1}{2x(1+x^2)}$ B) $\dfrac{1-2x\arctan x}{(1+x^2)^2}$ C) $\dfrac{2x\arctan x - 1}{(1+x^2)^2}$ D) $\dfrac{1}{2x(1+x^2)^2}$

E) $\dfrac{1}{(1+x^2)^2}$ F) $\dfrac{(1+x^2)\sec^2 x - 2x\arctan x}{(1+x^2)^2}$ G) $-\dfrac{2x}{(1+x^2)^2}$ H) $\dfrac{\sec^2 x}{2x}$

I) $x\arctan(1+x^2)$ J) $\dfrac{1}{(1+x^2)} - 2x$

For questions 2 - 11, please refer to the graphs of $f(x)$ and $g(x)$ given below. Questions 2 - 6 (worth 2 points each) are "fill in the blank", where you will select your answers from those listed below the graphs. Questions 7 - 11 (worth 2 point each) are true/false questions.





The "fill in the blank" answer choices are:

A) -4 B) -3 C) -2 D) -1 E) 0

F) 1 G) 2 H) 3 I) 4 J) ∞

2. $\lim_{x \to 6^-} \frac{g(x)}{f(x)} =$ _____.

3. $\lim_{x \to 0^+} \frac{f(x)}{g(x)} =$ _____.

4. $f''(-1.5) =$ _____.

5. $\lim_{x \to -3^+} g(x) =$ _____.

6. $\lim_{x \to -1} f(x) =$ _____.

The following 5 questions are true/false and still refer to the functions $f(x)$ and $g(x)$, whose graphs are shown on the previous page.

7. f is differentiable at $x = -1$.

A) True B) False

8. f is continuous at $x = -1$.

A) True B) False

9. $x = 4$ is a critical value for $g(x)$.

A) True B) False

10. $g''(x) < 0$ on $(-\infty, -3)$.

A) True B) False

11. $f'(x) > 0$ on $(-\infty, -3)$.

A) True B) False

12. Find the absolute minimum value of the function $f(x) = x^3 - 12x + 1$ on the interval $[-3, 5]$.

A) 0 B) -1 C) -2 D) -3 E) -9

F) -15 G) -18 H) -22 I) -25 J) -50

13. Find $\lim_{x \to 0} \dfrac{e^{4x} - 1}{2\sin(3x)}$.

A) ∞ B) 4 C) 2 D) $\frac{4}{3}$ E) 1

F) $\frac{1}{2}$ G) $\frac{2}{3}$ H) -1 I) 0 J) $-\pi$

14. Suppose a particle moving in a straight line has velocity function $v(t) = t - 2$ (in meters per second) for $0 \leq t \leq 6$. What is the particle's displacement?

A) 0 meters B) 4 meters C) 6 meters D) 8 meters E) 10 meters

F) 12 meters G) 16 meters H) 18 meters I) 24 meters J) 36 meters

15. What is the total distance travelled for the particle described in problem 14?

A) 0 meters B) 36 meters C) 24 meters D) 18 meters E) 16 meters

F) 14 meters G) 12 meters H) 10 meters I) 6 meters J) 2 meters

16. For what values of x is the function $f(x) = x^3 - x$ both **increasing and concave down**?

A) no values of x

B) $x < -\frac{1}{\sqrt{3}}$ and $x > \frac{1}{\sqrt{3}}$

C) $0 < x < \frac{1}{\sqrt{3}}$

D) $0 < x < \frac{1}{3}$

E) $x > 0$

F) $-\frac{1}{3} < x < \frac{1}{3}$

G) $-\frac{1}{\sqrt{3}} < x < 0$

H) $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

I) $x > \frac{1}{\sqrt{3}}$

J) $x < -\frac{1}{\sqrt{3}}$

Part II: These are the "free response" problems worth a total of 56 points. Write your answers on the test pages. <u>Show your work</u> neatly and cross out irrelevant scratch-work, false starts, etc.

Please put your NAME on each of the following pages, since they may be separated during grading. Remember to turn in your notecard with these pages if you wish to take part in the "Impress Me With Your Notecard . . . Win Great Prizes" contest.

Name: _____ **Discussion Section:** _____

17. (24 points) The theme of this question is integration.

a) Use the **definition** of the definite integral to evalute $\int_0^4 (x+1)\,dx$.

b) Evaluate $\displaystyle\int \frac{x^2 + \sqrt{x}\left(2^x + \sec^2 x\right) + 1}{\sqrt{x}}\, dx.$

c) Evaluate $\displaystyle\int \frac{1 + 6x}{\sqrt{2 + x + 3x^2}}\, dx.$

d) Evaluate $\displaystyle\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx.$

Name: _____ **Discussion Section:** _____

18. (21 points) The theme of the following problem is derivatives and equations of tangent lines.

a) (6 points) Differentiate $f(x) = e^{\sin(x^2)}$.

b) (5 points) Find $f'(x)$ if $f(x) = \ln\left(\dfrac{2^{x^2}\sin x}{x^6}\right)$. [Hint: There is an efficient solution to this problem, and a solution that makes for not very pleasant differentiation.]

c) (5 points) Find the equation of the tangent line to $x^3y^3 + x = 2$ at the point $(1, 1)$.

d) (5 points) Find the equation of the tangent line to $y = f(x) = \int_1^{x^2} \sqrt{1 + w^5}\, dw$ at $x = 1$.

Name: _____ **Discussion Section:** _____

19. (11 points) A potpourri of problems for your enjoyment. Remember to show all of your work.

a) (6 points) A ladder 50 feet long is leaning against a wall. The bottom of the ladder is sliding away from the wall at a constant rate of 2 ft/sec. At what speed is the top of the ladder sliding down the wall at the instant when the bottom of the ladder is 30 feet from the wall?

b) (5 points) Find the horizontal asymptotes of the function $f(x) = \cos\left(\dfrac{\pi x^4 - 2x + 1}{x^4 + 5x^2 + 10}\right)$.

20. Calculus is one of the great intellectual achievements of the human mind.

A) true B) maybe C) false
D) every other Thursday E) when pigs fly F) Yes, but that's an understatement.
G) What do you mean "one" of? H) duuh . . . I) Yes, along with Cheez Whiz.
J) Frankly, I think pluriharmonic and plurisubharmonic functions just don't get enough credit.