MATH 131 EXAMINATION I FALL 2001

This exam should have 20 questions. Part I will have 18 questions,13 multiple choice and 5 true and false. Part II will have two hand graded questions. Please check to see that your exam is complete. If you do not have a PENCIL to mark your card, please ask to borrow one from your proctor.

Write your social security number on the nine blank lines at the top of your answer card, using one blank for each digit. **Shade in the corresponding boxes below.** Print your name at the top of your card and ***sign*** *your name on the back.* Leave the boxes for version type blank.

As you work the exam, lightly shade in the correct answers on your answer card. At the end of your exam, when you are certain of all your choices, *darken* all your answer boxes. If your card becomes damaged please ask your proctor for a new one.

1. Find the largest number in the domain of the function

$$f(x) = \sqrt{9-(x+1)^2} \ .$$

A) 0

B) 1

C) 2

D) 3

E) 4

F) 5

G) 6

H) 7

I) 8

J) 9

$9-(x+1)^2 \geq 0$

$(x+1)^2 \leq 9$

$-3 \leq x+1 \leq 3$

$-4 \leq x \leq 2$

largest is $x=2$

2. Let $f(x) = x^2 - 5x$. The <u>slope</u> of the <u>secant line</u> from the point where $x = -1$ to the point where $x = 1$ is :

A) 0

B) 1

C) -1

D) 2

E) -2

F) 5

G) -5

H) 10

I) -10

J) slope is undefined

$$\text{slope} = \frac{f(1) - f(-1)}{1 - (-1)}$$

$$= \frac{-4 - (+6)}{2} = -5$$

3. Suppose $f(x) = \frac{2}{x+1}$, $g(x) = \sqrt{x+2}$ and $h(x) = x + 3$.

Find the value of $(g \circ h \circ f)(1)$.

A) 0

B) 1

C) $\sqrt{2}$

D) $\sqrt{3}$

E) 2

F) $\sqrt{5}$

G) $\sqrt{6}$

H) $\sqrt{7}$

I) $\sqrt{8}$

J) 3

$g(h(f(1)))$, $f(1) = 1$

$= g(h(1))$, $h(1) = 4$

$= g(4)$

$= \sqrt{6}$

4. The distance , d , travelled by a train is given by the following table , where t represents time in hours and d is measured in miles .

t	0	1	2	3	4	5
d	0	70	150	250	380	510

What is the <u>average velocity</u> of the train over the time period $2 \leq t \leq 5$?

A) 75 mph

B) 80 mph

C) 100 mph

D) 102 mph

E) 103 mph

F) 115 mph

G) 120 mph

H) 130 mph

I) 180 mph

J) 220 mph

$$\frac{d(5) - d(2)}{5 - 2}$$

$$= \frac{510 - 150}{3} = 120$$

5. The graph of g(x) is obtained from the graph of $f(x) = e^x$ by performing the following steps , in the order given :

(i) reflect the graph across the y-axis
(ii) compress the graph horizontally by a factor of 3
(iii) shift the graph 2 units up
(iv) stretch the graph vertically by a factor of 5

Which of the following is a formula for g(x) ?

A) $5 \cdot e^{-3x} + 10$

B) $-5 \cdot e^{3x} + 10$

C) $5 \cdot e^{-3x} + 2$

D) $e^{3x} - 10$

E) $5 \cdot e^{x} - 10$

F) $3 \cdot e^{-x} + 2$

G) $10 \cdot e^{x} - 5$

H) $e^{-3x} + 10$

I) $e^{-3x} + 2$

J) $3 \cdot e^{-5x} + 10$

(i) $f(x) = e^{-x}$

(ii) $f(x) = e^{-3x}$

(iii) $f(x) = e^{-3x} + 2$

(iv) $f(x) = 5(e^{-3x} + 2)$

$= 5e^{-3x} + 10$

6. Let $f(x) = \frac{x}{x+1}$ and $g(x) = x + a$, where a is some constant. For what value of a does the graph of $f \circ g$ have an <u>x-intercept</u> at $x = 9$?

A) 15

B) 12

C) 9

D) 4

E) 1

F) -3

G) -5

H) -7

I) -9

J) -15

$$f(g(x)) = f(x+a)$$

$$= \frac{x+a}{x+a+1}$$

<u>Given</u>: $(9,0)$ is point on graph

$$0 = \frac{9+a}{9+a+1} \Rightarrow a = -9$$

7. Suppose $\lim_{x \to a} f(x) = L$. Then which of the following statements is guaranteed to be true ?

p : $\lim_{x \to a^-} f(x) = L$

q : $\lim_{x \to a^+} f(x) = L$

r : $f(a) = L$

A) p , q and r

B) only p and r

C) only p and q

D) only q and r

E) only p

F) only q

G) only r

H) none of them

I)

J)

8. If $G(X) = \frac{x+2}{3x-1}$, then an expression for $G^{-1}(x)$ is :

A) $\frac{x+2}{3x-1}$

B) $\frac{3x-1}{x+2}$

C) $\frac{x-2}{3x+1}$

D) $\frac{\frac{x}{2}+1}{x-3}$

E) $\frac{5x+2}{1-4x}$

F) $\frac{x-2}{3x-1}$

G) $\frac{10x-1}{x+2}$

H) $\frac{x+2}{x-\frac{1}{3}}$

I) $\frac{x+1}{x-1}$

J) G has no inverse

$$y = \frac{x+2}{3x-1}$$

$$3xy - y = x+2$$

$$3xy - x = y+2$$

$$x(3y-1) = y+2$$

$$x = \frac{y+2}{3y-1}$$

or

$$G^{-1}(x) = \frac{x+2}{3x-1}$$

9. The curve with parametric equations

$$x(t) = \ln(3t - 2) \quad , \quad y(t) = \ln[(6t - 4)^2]$$

has a <u>y-intercept</u> , y = :

A) 0

B) 1

C) 2

D) ln(2)

E) ln(4)

F) $\ln(\frac{9}{2})$

G) 3

H) 1.5

I) $\frac{7}{5}$

J) does not exist

y intercept mean $x=0$,

hence $0 = \ln(3t-2)$

$3t-2 = 1$

$3t = 3$

$\boxed{t=1}$

Then $y = \ln[(6-4)^2]$

$= \ln(4)$

10. Evaluate the function $f(x) = \frac{1-\sqrt{x}}{1-x}$ at values close to $x = 1$ to guess at the value of $\lim_{x \to 1} f(x)$.

A) 0

B) 1

C) -1

D) 0.5

E) -0.5

F) 0.25

G) -0.25

H) $\sqrt{2}$

I) $-\sqrt{2}$

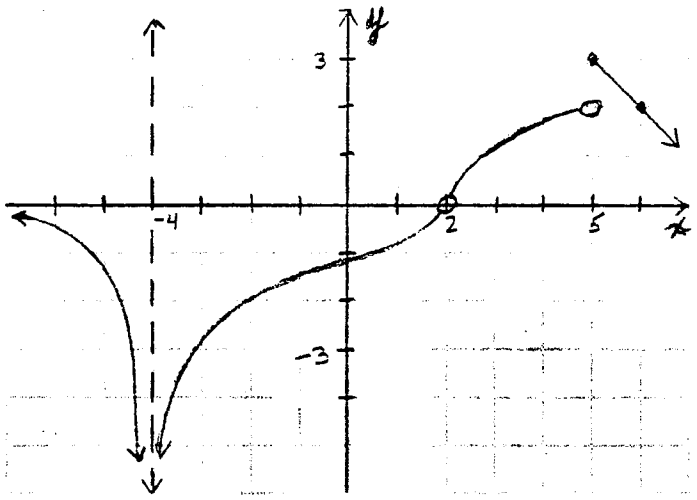
J) The limit does not exist

x	$f(x)$
.9	.513
1.1	.488
.999	.500
1.001	.499

Guess

$\lim_{x \to 1} f(x) = .5$

11. Consider the following graph of the function f(x) :



Which of the following statements is true ?

p: $\lim_{x \to -4}$ f(x) exists (no) f(x) heading towards $-\infty$

q: $\lim_{x \to 2}$ f(x) exists (yes) $\lim_{x \to 2} f(x) = 0$

r: $\lim_{x \to 5}$ f(x) exists (no) $\lim_{x \to 5^-} f(x) = 2$, $\lim_{x \to 5^+} f(x) = 3$ } not the same

A) p , q and r

B) only p and r

C) only p and q

D) only q and r

E) only p

F) only q

G) only r

H) none of them

12. The half-life of a certain isotope is 8 <u>days</u> (so half of any quantity of the isotope disintegrates in 8 days) . If the sample initially has a mass of 100 g , find an expression for the amount ,A(t), that remains after t <u>years</u> .

A) $100 \cdot 2^{-\frac{t}{8}}$

B) $100 \cdot 2^{t}$

C) $100 \cdot 2^{\frac{t}{8}}$

D) $100 \cdot 2^{-365t}$

E) 2^{-365t}

F) 100^{-2t}

G) $100 \cdot 2^{-\frac{365t}{8}}$

H) $100 \cdot e^{\frac{365t}{8}}$

I) $100 \cdot e^{-\frac{t}{8}}$

J) e^{-365t}

If ℓ = # of days

then

$$A(\ell) = 100\left(\tfrac{1}{2}\right)^{\ell/8}$$
$$= 100(2)^{-\frac{\ell}{8}}$$

t years $= 365\,t$ days

so $\ell = 365t$

and $A(t) = 100(2)^{-\frac{365t}{8}}$

13. Consider the following set of parametric equations:

$$x(t) = e^{\sin(t)} \quad , \quad y(t) = \cos(t) \quad .$$

What is the biggest value of the constant b such that the entire graph can be viewed in the window

$$b \le x \le e \quad , \quad -1 \le y \le 1 \quad ?$$

A) $b = \frac{1}{2}$

B) $b = \frac{1}{3}$

C) $b = 0$

D) $b = 1$

E) $b = \frac{1}{e}$

F) $b = e^2$

G) $b = -1$

H) $b = -2$

I) $b = -e$

J) $b = \frac{3}{4}$

On calculator it look like furthest point to the left on graph is $\sim (.3723, 0)$

$\frac{1}{e} = .367879$

14. True or False: If both $f(x)$ and $g(x)$ have as their <u>domain</u> the domain, D, then the <u>domain</u> of the function $f + g$ is also D.

A) True

B) False

15. True or False: In computing $\lim_{x \to a} f(x)$, the value of the function at $x = a$, $f(a)$, is irrelevant.

A) True

B) False

16. True or False: The graph of $f(x) = -e^{-x} + 1000$ never crosses the line $y = 781$.

A) True

B) False

solve $e^{-x} = 219$

$-x = \ln(219)$

$x = -\ln(219)$ "does cross"

or $\boxed{f(-\ln(219)) = 781 \quad = -219 + 1000 = 781}$

17. True or False: A curve in the xy - plane can have more than one set of parametric equations that describe it .

A) True

B) False

18. True or False: If f(t) is a function which gives the height of a football t seconds after kickoff then f(t) has an inverse function .

A) True

B) False

f(t) is not 1-1 since it reaches certain heights twice, once up + once down

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Part II: (25% of test points) In each problem, clearly show your solution in the space provided. Present a readable, orderly sequence of steps showing how you got the answer. Generally, a correct answer without supporting work might not recieve full credit.

Please put your name and ID number above and on each following sheet of Part II, in case the sheets get separated.

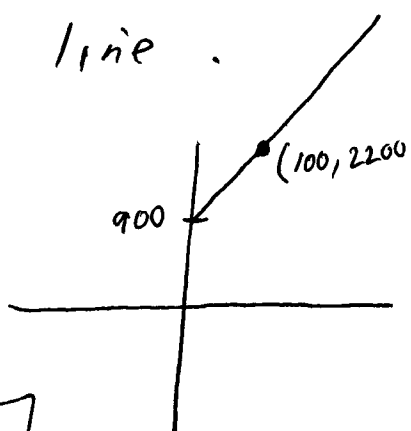
19. The manager of a furniture factory finds that it costs $2200 to manufacture 100 chairs in one day, and $4800 to manufacture 300 chairs in one day.

a) If we assume that the cost, y, as a function of the number of chairs, x, (produced in one day) is a linear function then write out the function y = f(x) and graph it (6 pts.).

Given $f(100) = 2200$ + $f(300) = 4800$

also $f(x)$ is a line.

Slope $= m = \dfrac{4800 - 2200}{300 - 100} = 13$



$$\boxed{y - 2200 = 13(x - 100)}$$

or $$\boxed{y = 13x + 900}$$

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19(cont).

b) Write out precisely what the slope of the graph represents (2 pts.) .

slope represents ratio of change in cost per change in amount of chairs produced in one day

or rise in cost per extra chair produced

i.e. \$13 for 1 extra chair produced that day.

c) What does the y-intercept of the graph represent (2 pts)?

y-intercept is 900 which means that it cost \$900 to run factory even if no chairs produced that day.

i.e. fixed daily costs of operating factory

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20. This problem concerns the <u>exponential function</u> $f(x) = C\,a^x$, where C and a are constants.

a) Suppose we know that the points $(0,-2)$ and $(-1,-6)$ lie on the curve $y = f(x)$. Calculate the <u>slope</u> of the <u>secant line</u> between these two points (2 pts.).

$$\text{Slope} = \frac{-6-(-2)}{-1-0} = 4$$

b) Using the fact that $(0,-2)$ and $(-1,-6)$ lie on the curve, determine the <u>values</u> of the constants C and a (i.e. Write out explicit formula for $f(x)$) . (4 pts.)

$$x=0,\ y=-2 \Rightarrow \boxed{-2 = C\,a^0 = C}$$

$$x=-1,\ y=-6 \Rightarrow -6 = -2a^{-1} = -\frac{2}{a}$$

$$\Rightarrow \boxed{a = \frac{1}{3}}$$

<u>or</u> $$f(x) = -2\cdot\left(\frac{1}{3}\right)^x = -2\,(3)^{-x}$$

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20(cont)

c) Using your answer from (b) on the previous page, write down a formula for $f^{-1}(x)$, as well as the <u>domain</u> and <u>range</u> of both $f(x)$ and $f^{-1}(x)$ (5 pts.) .

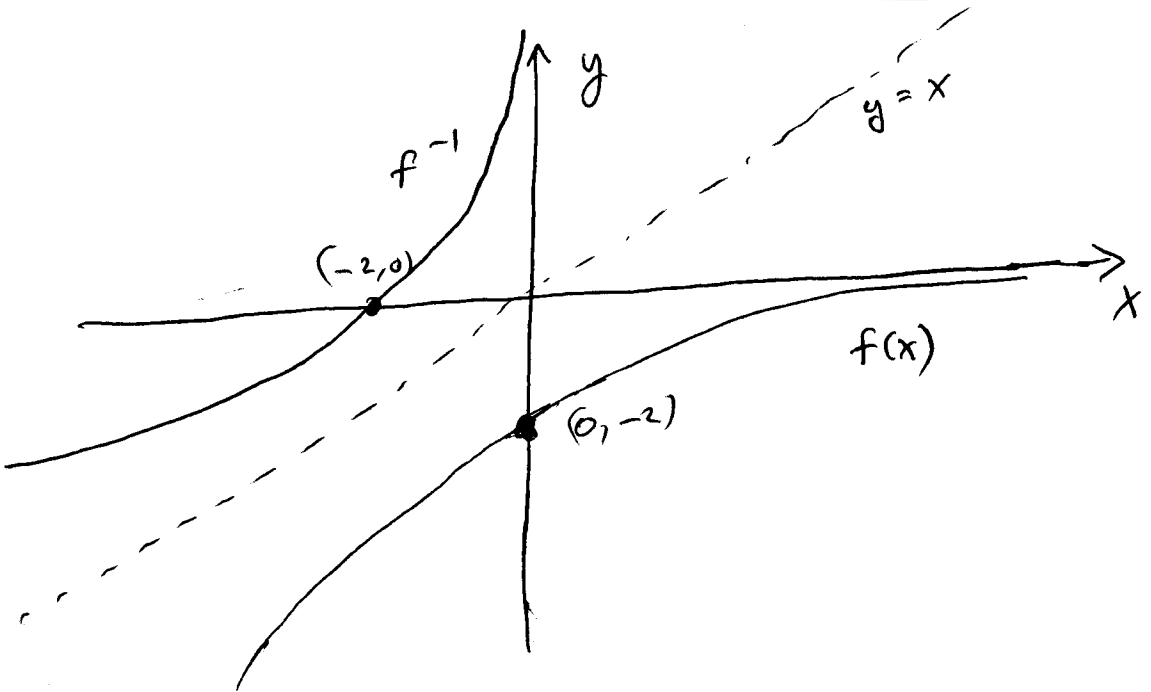
$$y = -2\left(\tfrac{1}{3}\right)^x \Rightarrow \left(\tfrac{1}{3}\right)^x = -\frac{y}{2}$$

$$\Rightarrow x = \log_{\frac{1}{3}}\left(-\frac{y}{2}\right)$$

$$\boxed{f^{-1}(x) = \log_{1/3}\left(\frac{-x}{2}\right)}$$

Domain of $f(x)$ is all x = Range of f^{-1}

Range of $f(x)$ is all negative numbers = Domain of f^{-1}

d) Sketch the graphs of f and f^{-1} on the axes below .

Please include the coordinates of all <u>intercepts</u> (4 pts.).