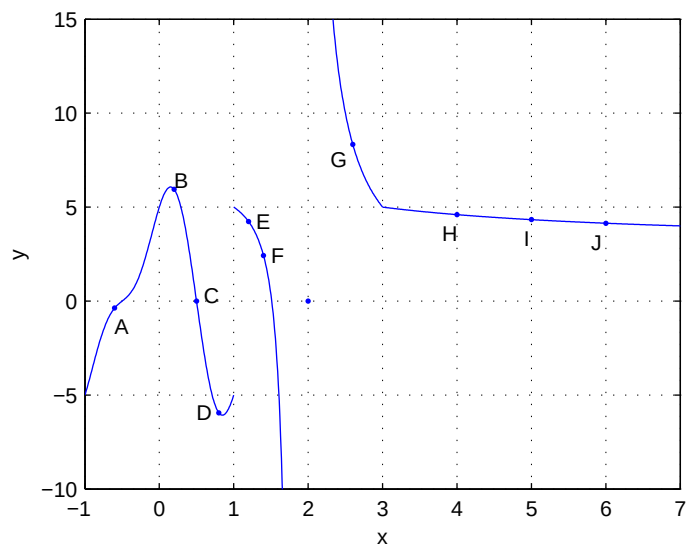
EXAM 1, MATH 131
WEDNESDAY, FEBRUARY 8, 2006

Problems 1 through 14 are multiple choice and worth five points apiece. Problems 15 through 17 are hand-graded and worth ten points apiece. There is a total of 100 points for the whole examination. Do all work in the exam booklet. You may use a scientific calculator, but NOT a graphing calculator.

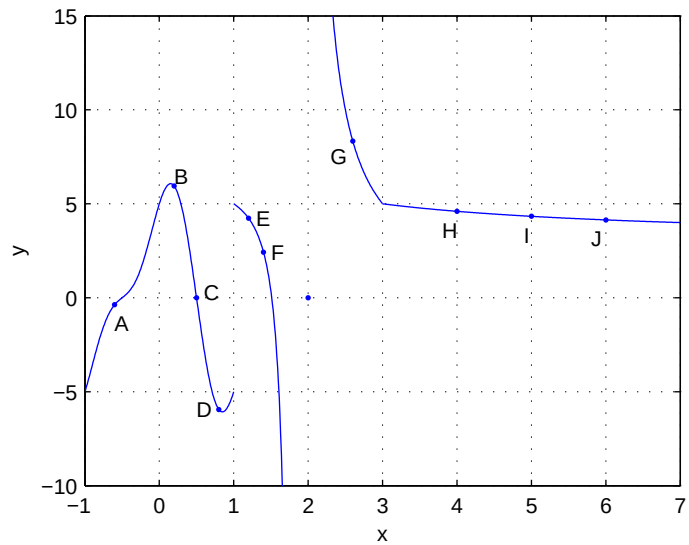
(1) The graph of the function $y = f(x)$ is



Find $\lim_{x\to 1^+} f(x)$.

(A) 15
(B) 10
(C) 5
(D) 0
(E) -5
(F) -10
(G) -15
(H) $-\infty$
(I) ∞
(J) Does not exist.

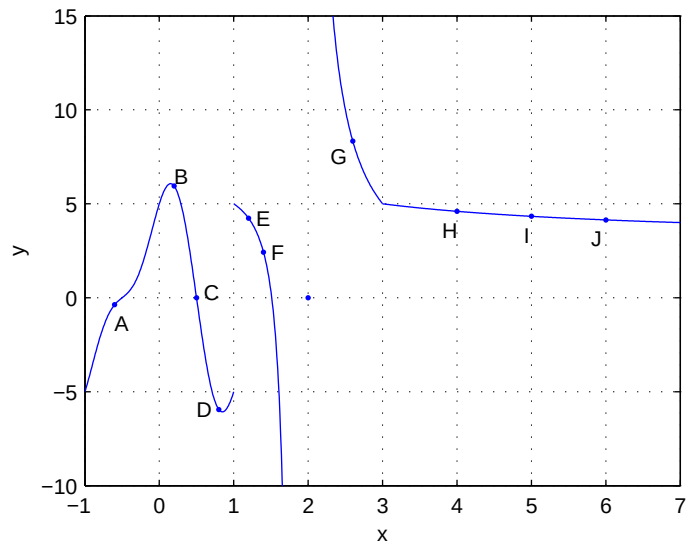
(2) The graph of the function $y = f(x)$ is



Find the point where the instantaneous rate of change of y with respect to x is greatest. (Be careful of the sign).

(A) A
(B) B
(C) C
(D) D
(E) E
(F) F
(G) G
(H) H
(I) I
(J) J

(3) The graph of the function $y = f(x)$ is

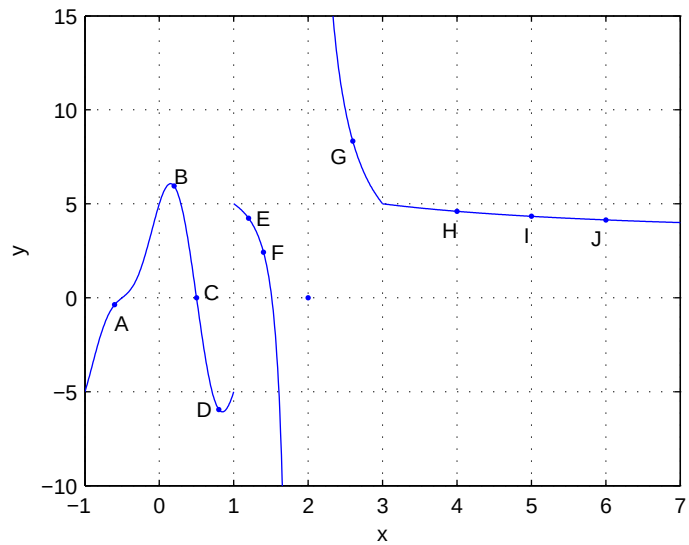


Find

$$\lim_{x \to 3} f(x)$$

(A) Does not exist.
(B) 1
(C) 2
(D) 3
(E) 4
(F) 5
(G) 6
(H) 7
(I) 8
(J) 9

(4) The graph of the function $y = f(x)$ is



Find what the graph indicates is

$$\lim_{x \to 2^-} f(x)$$

(A) -3
(B) -2
(C) -1
(D) 0
(E) 1
(F) 2
(G) 3
(H) 4
(I) ∞
(J) $-\infty$

(5) A point moves along a path according to the parametric equations

$$x = \cos 3t, \quad y = \sin 3t, \quad 0 \leq t \leq 2\pi$$

Find the number of times the point hits the y-axis.

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5
(F) 6
(G) 7
(H) 8
(I) 9
(J) 10

(6) Which of the following parametric equations traces a circle with center $(-1, 3)$, radius 2, and traversed exactly once in the counterclockwise direction?

(a) $x = -1 + 2\cos t$, $y = 3 + 2\sin t$, for $0 \leq t \leq 2\pi$.
(b) $x = -1 + 2\cos t$, $y = 3 + 2\sin t$, for $0 \leq t \leq \pi$.
(c) $x = -\cos 2t$, $y = 3\sin 2t$, for $0 \leq t \leq \pi$.
(d) $x = -1 + 2\cos t$, $y = 3 + 2\sin t$, for $\pi/2 \leq t \leq 5\pi/2$.
(e) $x = -1 + 2\cos t$, $y = 3 - 2\sin t$, for $0 \leq t \leq 2\pi$.

(A) a and b
(B) a and c
(C) a and d
(D) a and e
(E) b and c
(F) b and d
(G) b and e
(H) c and d
(I) c and e
(J) d and e

(7) The point $P(1, \frac{1}{2})$ lies on the graph of $y = \frac{x}{1+x}$. If Δx is a small non-zero number, let Q be the point on the graph whose first coordinate is $1 + \Delta x$. Find the slope of the secant line PQ.

(A) $\frac{1}{1+\Delta x}$
(B) $\frac{1}{2+\Delta x}$
(C) $\frac{1}{3+\Delta x}$
(D) $\frac{1}{4+\Delta x}$
(E) $\frac{1}{5+\Delta x}$
(F) $\frac{2}{1+\Delta x}$
(G) $\frac{1}{2(2+\Delta x)}$
(H) $\frac{1}{2(3+\Delta x)}$
(I) $\frac{1}{2(4+\Delta x)}$
(J) $\frac{1}{2(5+\Delta x)}$

(8) A tank holds 1000 gallons of water. A plug in the bottom of the tank is pulled at time $t = 0$. The values in the table show the volume V of water (in gallons) remaining in the tank after t minutes.

t(min)	5	10	15	20	25	30
V(gal)	694	444	250	111	28	0

Let A be the average rate of change of V over the interval $10 \leq t \leq 15$ and let

$$B = \frac{V(15 + \Delta t) - V(15)}{\Delta t}, \text{ for } \Delta t = -5$$

Find the pair of numbers (A, B).

(A) $(\frac{194}{5}, -\frac{111}{5})$
(B) $(-\frac{111}{5}, -\frac{222}{5})$
(C) $(\frac{111}{5}, \frac{111}{5})$
(D) $(-\frac{194}{5}, -\frac{194}{5})$
(E) $(\frac{222}{5}, -\frac{222}{5})$
(F) $(-\frac{222}{5}, -\frac{194}{5})$
(G) $(\frac{194}{5}, \frac{194}{5})$
(H) $(-\frac{194}{5}, -\frac{139}{5})$
(I) $(\frac{333}{5}, -\frac{111}{5})$
(J) $(-\frac{333}{5}, \frac{222}{5})$

(9) The function $y = f(x)$ is defined on $x > 0$ by

$$f(x) = \begin{cases} \frac{x^2-2x}{x^2-x-2} & \text{if } x \neq 2 \\ L & \text{if } x = 2 \end{cases}$$

Find the value of L for which f will be continuous at $x = 2$.

(A) 1/12
(B) 1/6
(C) 1/4
(D) 5/12
(E) 1/2
(F) 7/12
(G) 2/3
(H) 3/4
(I) 5/6
(J) 11/12

(10) The graph of $y = \sqrt{4 - x^2}$ is the upper half of the circle $x^2 + y^2 = 4$. Find the slope of the tangent line to this circle at the point $(1, \sqrt{3})$.

(A) -1
(B) $-\frac{1}{\sqrt{2}}$
(C) $-\frac{1}{\sqrt{3}}$
(D) $\frac{1}{2}$
(E) $-\frac{1}{\sqrt{5}}$
(F) $-\frac{1}{\sqrt{6}}$
(G) $-\frac{1}{\sqrt{7}}$
(H) $\sqrt{3}$
(I) $-\frac{1}{3}$
(J) 0

(11) A ball's height t seconds after being dropped from a 600 ft tower is $h = 600 - 16t^2$ ft. Find its **average velocity** over the period from 1 second to 2 seconds.

(A) 41 ft/sec
(B) -42 ft/sec
(C) 43 ft/sec
(D) -44 ft/sec
(E) 45 ft/sec
(F) -46 ft/sec
(G) 47 ft/sec
(H) -48 ft/sec
(I) 49 ft/sec
(J) -50 ft/sec

(12) A ball's height t seconds after being dropped from a 600 ft tower is $h = 600 - 16t^2$ ft. Find its **instanteneous veclocity** at $t = 2$ seconds.

(A) 44 ft/sec
(B) -48 ft/sec
(C) 52 ft/sec
(D) -56 ft/sec
(E) 60 ft/sec
(F) -64 ft/sec
(G) 68 ft/sec
(H) -72 ft/sec
(I) 76 ft/sec
(J) -80 ft/sec

(13) Find the slope of the tangent line to the graph of $y = 600 - 16x^2$ at the point where $x = 2$.

(A) 44
(B) -48
(C) 52
(D) -56
(E) 60
(F) -64
(G) 68
(H) -72
(I) 76
(J) -80

(14) Find the horizontal asymptotes, if it has any, of

$$\frac{x^3 + 5x}{2x^3 - x^2 + 4}$$

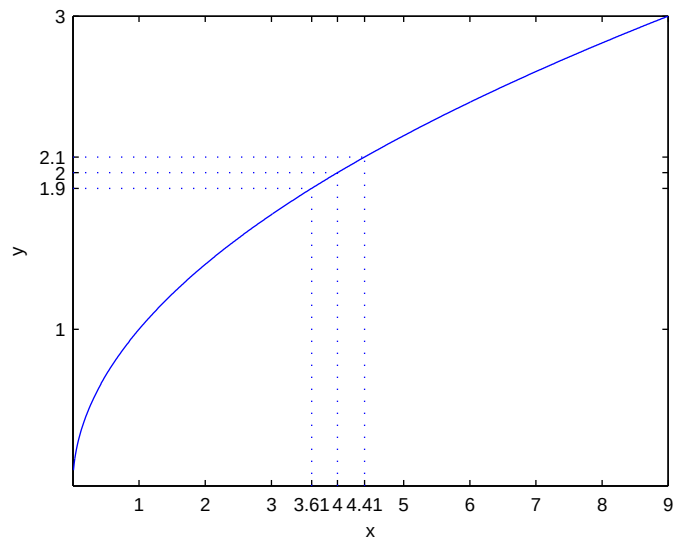
(A) $y = 0$
(B) $y = 1/4$
(C) $y = 1/3$
(D) $y = 1/2$
(E) $y = 1$
(F) $y = 0$ and $y = 5/4$
(G) $y = 4/3$ and $y = 1/2$
(H) $y = 3/2$ and $y = 1/3$
(I) $y = 7/4$ and $y = 4/3$
(J) No horizontal asymptotes.

Hand graded problems begin here. On these show all your work and indicate clearly your answer to the problem. Partial credit will be given for partially completed solutions, provided the work is neat and the grader does not have to work too hard figuring out what you did.

(15) Sketch the graph of a function $y = f(x)$ that is continuous at every point except $x = 0$ and $x = 2$ and for which $\lim_{x\to 2} f(x) = -\infty$, $\lim_{x\to\infty} f(x) = \infty$, $\lim_{x\to-\infty} f(x) = 0$, $\lim_{x\to 0^+} f(x) = \infty$, and $\lim_{x\to 0^-} f(x) = -\infty$.

(16) (a) State the definition of the derivative of the function $y = f(x)$ at the point $x = a$. (Note: the correct answer is not just $f'(a)$, but rather the definition of $f'(a)$).

(b) The definition of $\lim_{x\to 4}\sqrt{x} = 2$, is: given $\epsilon > 0$, there exists a $\delta > 0$ such that if $|x-4| < \delta$, then $|\sqrt{x} - 2| < \epsilon$. The following graph of $y = \sqrt{x}$ shows the situation for $\epsilon = .1$. Find the largest value of δ that will satisfy the definition for this value of ϵ.



(17) Find

$$\lim_{x \to 7} \frac{\sqrt{x+2} - 3}{x - 7}$$