PART I consists of 12 multiple choice questions (worth 5 points each) for a total of 60 points. Mark the correct answer on the answer card. For part I only the answer on the card will be graded.

PART II consists of 4 hand graded problems (worth 10 points each) for a total of 40 points. A correct answer without supporting work may get no credit. Present a readable, orderly sequence of steps showing how you got your answer.

Part I (80 points):

1) Find an equation for the tangent line to the curve \( y = x \cos(x) \) at \((\pi, -\pi)\).

   A) \( y = x \)
   B) \( x + y = 0 \)
   C) \( 2x - y = \pi \)
   D) \( y = 3x - \pi \)
   E) \( x + 2y = \frac{3\pi}{2} \)
   F) \( x - y = 2\pi \)
   G) \( y = \pi x - \pi \)
   H) \( y = \pi x + \pi \)
   I) \( x - 2\pi y = 2 \)
   J) \( 3x - 2y = \frac{\pi}{2} \)

2) Find the slope of the tangent line to the curve \( x^2 - 2xy + y^3 = 1 \) at \((2, 1)\).

   A) 1  B) -1  C) 2  D) -2  E) 3  F) -3  G) \( \frac{3}{3} \)
   H) \( -\frac{2}{3} \)
   I) \( \frac{2}{3} \)
   J) \( -\frac{3}{2} \)
3) If \( f(x) = \frac{1}{\sqrt{x^2 + x + 2}} \) then find \( f'(1) \).

A) 0  B) -2  C) \( \frac{3}{2} \)  D) -\( \frac{3}{18} \)  E) \( \frac{1}{4} \)  F) -\( \frac{3}{17} \)  G) \( \frac{3}{4} \)  H) -\( \frac{3}{16} \)  I) \( \frac{3}{2} \)  J) -\( \frac{1}{2} \)

4) For the curve given by the parametric equations \( x = t \sin(t) \), \( y = t \cos(t) \), find the value of \( \frac{dy}{dx} \) at the point \( (0, -\pi) \), where \( \frac{\pi}{2} \leq t \leq \frac{3\pi}{2} \).

(From \( x = 0 \) and \( y = -\pi \) you need to find the value of \( t \) in the above domain.)

A) \( \pi \)  B) -\( \pi \)  C) \( 2\pi \)  D) -\( 2\pi \)  E) \( \frac{\pi}{2} \)  F) -\( \frac{\pi}{2} \)  G) \( \frac{1}{\pi} \)  H) -\( \frac{1}{\pi} \)  I) \( \frac{3\pi}{2} \)  J) -\( \frac{3\pi}{2} \)
5) If $f(x) = (\tan^{-1}(x))^2$ then $f'(1) = :$

A) $\frac{\pi}{6}$
B) $\frac{\pi}{3}$
C) $\frac{\pi}{4}$
D) $\frac{\pi}{8}$
E) $\frac{\pi}{2}$
F) $\frac{\pi}{12}$
G) $\frac{\pi}{9}$
H) $\pi$
I) $2\pi$
J) 0

6) Solve for $x$ in the equation $e^{2x+3} - 7 = 0$.

A) $\frac{\ln(7)+3}{2}$
B) $\frac{\ln(3)+2}{7}$
C) $\frac{\ln(2)+7}{3}$
D) $\frac{\ln(3)+3}{7}$
E) $\frac{\ln(7)+2}{3}$
F) $\frac{\ln(7)-3}{2}$
G) $\frac{\ln(3)-2}{7}$
H) $\frac{\ln(2)-7}{3}$
I) $\frac{\ln(7)-2}{3}$
7) Find \( L(x) \), the **linear approximation** of the function \( f(x) = \sqrt{4+2x} \) at \( a = 0 \) and use it to approximate the number \( \sqrt{4.08} \).

A) \( 2 + \frac{1}{10} \)  
B) \( 2 + \frac{1}{15} \)  
C) \( 2 + \frac{1}{20} \)  
D) \( 2 + \frac{1}{25} \)  
E) \( 2 + \frac{1}{30} \)  
F) \( 2 + \frac{1}{35} \)  
G) \( 2 + \frac{1}{40} \)  
H) \( 2 + \frac{1}{45} \)  
I) \( 2 + \frac{1}{50} \)  
J) \( 2 + \frac{1}{55} \)  

8) If \( f(x) = xe^x \) then find \( f'(1) \).

A) 0  
B) 1  
C) \(-1\)  
D) 2  
E) \(-2\)  
F) \(e\)  
G) \(-e\)  
H) \(1/e\)  
I) \(-1/e\)  
J) Derivative DNE
9) If \( f(x) = x \cdot \ln(e^{\sqrt{x}}) \), find \( f'(1) \).

- A) \( \frac{1}{2} \)
- B) \( 1 \)
- C) \( e \)
- D) \( \frac{2}{3} \)
- E) \( \frac{5}{2} \)
- F) \( 2 \)
- G) \( \frac{3}{2} \)
- H) \( \frac{1}{e} \)
- I) \( \frac{1}{e} \)
- J) \( 2e \)

10) The slope of the tangent line to the curve \( x \cdot \arctan(y) + x \cdot y = \frac{\pi + 4}{4} \) at the point \( (1, 1) \) is equal to:

- A) \( \frac{2}{3} \left( \arctan(1) + 1 \right) \)
- B) \( -\frac{2}{3} \left( \arctan(1) + 1 \right) \)
- C) \( \frac{1}{2} \left( \arctan(1) + 2 \right) \)
- D) \( -\frac{1}{2} \left( \arctan(1) + 2 \right) \)
- E) \( \frac{2}{3} \left( \arctan(1) + 4 \right) \)
- F) \( -\frac{2}{3} \left( \arctan(1) + 4 \right) \)
- G) \( \frac{1}{2} \left( \arctan(1) + 4 \right) \)
- H) \( -\frac{1}{2} \left( \arctan(1) + 4 \right) \)
- I) \( \frac{1}{4} \left( \arctan(1) + 2 \right) \)
- J) \( -\frac{1}{4} \left( \arctan(1) + 2 \right) \)
11) Eliminate the parameter to find a **Cartesian equation** for the curve
\[ x = -1 + 3 \sec(t) \quad y = 2 + 3 \tan(t) \]

A) \( (x - 1)^2 + (y - 2)^2 = 3 \)
B) \( (x - 1)^2 - (y - 2)^2 = 3 \)
C) \( (x + 1)^2 + (y - 2)^2 = 3 \)
D) \( (x + 1)^2 - (y - 2)^2 = 3 \)
E) \( (x - 1)^2 + (y - 2)^2 = 9 \)
F) \( (x - 1)^2 - (y - 2)^2 = 9 \)
G) \( (x + 1)^2 + (y - 2)^2 = 9 \)
H) \( (x + 1)^2 - (y - 2)^2 = 9 \)
I) \( (x + 1)^2 + (y + 2)^2 = 9 \)
J) \( (x + 1)^2 + (y + 2)^2 = 9 \)
12) The length of a rectangle is increasing at a rate of 4 cm/s and its width is decreasing at the rate of 3 cm/s. When the length is 20 cm and the width is 10 cm, what is the rate of change of the area?

A) increasing at 5 cm²/s
B) decreasing at 5 cm²/s
C) increasing at 10 cm²/s
D) decreasing at 10 cm²/s
E) increasing at 15 cm²/s
F) decreasing at 15 cm²/s
G) increasing at 20 cm²/s
H) decreasing at 20 cm²/s
I) increasing at 25 cm²/s
J) decreasing at 25 cm²/s
PART II: Show the work you did to get the answer in a readable orderly form.

13) In each case below write a formula for $f'(x)$. You don't have to simplify it.

   a) $f(x) = \sin^{-1}(\tan(x))$. (3 pts)

   b) $f(x) = x \cdot 4^{-x^2}$. (3 pts)

   c) $f(x) = \sqrt{\frac{x^2 + 1}{x^2 - 1}}$. (4 pts)
PART II: Show the work you did to get the answer in a readable orderly form.

14) The curve \( y^2 = x^3 + 3x^2 \) is called the Tschirnhausen cubic.

a) Find an equation of the tangent line to this curve at \((1, -2)\). (5 pts)

b) Find the \( x \) & \( y \) coordinates of the only two points on the curve at which the tangent line is horizontal. (5 pts)
Name: __________________________ ID number___________________

PART II: Show the work you did to get the answer in a readable orderly form.

15) If a bacteria population starts with 200 bacteria and triples every 4 hours, then the number of bacteria after \( t \) hours is given by the formula

\[
n = 200 \cdot 3^{\frac{t}{4}}
\]

a) Find the inverse of this function, describing \( t \) as a function of \( n \). (5 pts)

b) At what rate are the bacteria increasing 8 hours after the start? (5 pts)
Name: ___________________________  ID number ____________________

PART II: Show the work you did to get the answer in a readable orderly form.

16) At 2:00 PM sailboat B is 4 km south of sailboat A. After that A starts moving east at 4 km/hr and B starts moving east at 1 km/hr.

a) Draw a diagram of the movement of the two boats and calculate the distance between the two boats at 3:00 PM. (3 pts)

b) What is the rate of change of the distance between the two boats at 3:00 PM? (7 pts)