Problems 1 through 14 are multiple choice and worth five points apiece. Problems 15 through 17 are hand-graded and worth ten points apiece. There is a total of 100 points for the whole examination. Do all work in the exam booklet. You may use a scientific calculator, but NOT a graphing calculator.

(1) Find the pairs that match the graph of each function in (a)-(d) with the graph of its derivative in I-IV.

(A) (a, I), (b, II), (c, III), (d, IV)
(B) (a, I), (b, II), (c, IV), (d, III)
(C) (a, II), (b, I), (c, III), (d, IV)
(D) (a, II), (b, I), (c, IV), (d, III)
(E) (a, III), (b, I), (c, II), (d, IV)
(F) (a, III), (b, I), (c, IV), (d, II)
(G) (a, I), (b, III), (c, II), (d, IV)
(H) (a, I), (b, III), (c, IV), (d, II)
(I) (a, II), (b, IV), (c, III), (d, I)
(J) (a, II), (b, IV), (c, I), (d, III)

Easiest to see that (b) $\leftrightarrow$ IV.
Only answers (I) and (J) have the pair (b, IV). Next see that (c) $\leftrightarrow$ I (and not with III).
(2) If \( y = x^3 - 4x + 6 \), find the instantaneous rate of change of \( y \) with respect to \( x \) when \( x = 2 \).

(A) Does not exist.

(B) 1

(C) 2

(D) 3

(E) 4

(F) 5

(G) 6

(H) 7

(I) 8

(J) 9

\[ y' = 3x^2 - 4 \]

\[ y'(2) = 3 \cdot 2^2 - 4 = 12 - 4 = 8 \]
3. Find

\[ \lim_{h \to 0} \frac{e^h - 1}{h} = 1, \text{ by definition of } e. \]
(4) Find the slope of the tangent line at \( x = 2 \) of the graph of

\[
g(x) = \frac{3x - 1}{2x + 1}
\]

(A) 1/10
(B) 1/9
(C) 1/8
(D) 1/7
(E) 1/6
(F) 1/5
(G) 1/4
(H) 1/3
(I) 1/2
(J) 1

\[
g'(x) = \frac{(2x+1)\cdot 3 - (3x-1)\cdot 2}{(2x+1)^2}
\]

\[
= \frac{6x+3 - 6x + 2}{(2x+1)^2} = \frac{5}{(2x+1)^2}
\]

\[
g'(2) = \frac{5}{5^2} = \frac{1}{5}
\]
(5) Suppose that $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$, and $g'(5) = 2$. Find $(f \cdot g)'(5)$.

(A) -16  
(B) -15  
(C) -14  
(D) -13  
(E) -12  
(F) 11   
(G) 10   
(H) 9    
(I) 8    
(J) 7    

$$(f \cdot g)'(s) = f'(s)g(s) + f(s)g'(s), \text{ product rule}$$

$$= 6 \cdot (-3) + 1 \cdot 2 = -18 + 2 = -16$$
(6) A particle's position on a number line at time \( t \) seconds is \( s = t^3 - 12t^2 + 36t \) meters. Find the time when its acceleration is 0.

\[
S' = 3t^2 - 24t + 36 \\
\text{accel} = S'' = 6t - 24
\]

0 when \( t = \frac{24}{6} = 4 \)

(A) 0  
(B) 1  
(C) 2  
(D) 3  
(E) 4  
(F) 5  
(G) 6  
(H) 7  
(I) 8  
(J) 9
(7) Find

\[ \lim_{x \to 0} \frac{\sin x}{x} = 1 \]

(A) 0
(B) \(\pi/6\)
(C) \(\pi/4\)
(D) \(\pi/3\)
(E) \(\pi/2\)
(F) 1
(G) 1/2
(H) e
(I) \(e^{-1}\)
(J) \(\sqrt{3}/2\)
(8) For \( y = e^{x \cos x} \), find \( \frac{dy}{dx} \).

(A) \( e^{x \cos x} \cos x \)

(B) \( e^{x \cos x} x \sin x \)

(C) \( e^{x \cos x} (-x \sin x) \)

(D) \( e^{x \cos x} (\cos x - x \sin x) \)

(E) \( e^{x \cos x} - \cos x \)

(F) \( \cos x - x \sin x \)

(G) \( e^{\cos x} \)

(H) \( e^{x \cos x} (\cos x + x \sin x) \)

(I) \( -x \cos x \)

(J) \( \cos x + x \sin x \)

\[ \text{Chain rule } \Rightarrow \]

\[ (e^{x \cos x})' = e^{x \cos x} \cdot (x \cos x)' \]

\[ = e^{x \cos x} (\cos x - x \sin x) \]
(9) If $f$ is the function whose graph is the solid curve, and if $g$ is the function whose graph is the dashed curve, let $u(x) = f(g(x))$. Find $u'(1)$.

\[
\begin{align*}
\text{Chain rule: } & \quad u'(1) = f'(g(1)) \cdot g'(1) \\
\text{ } & \quad g(1) = 3 \\
\text{ } & \quad f'(3) = \frac{-1}{4} \\
\text{ } & \quad g'(1) = -\frac{6}{2} = -3 \\
\text{ } & \quad u'(1) = \left(\frac{-1}{4}\right) \cdot (-3) = \frac{3}{4}
\end{align*}
\]
(10) A mass on a spring vibrates horizontally on a smooth level surface. Its equation of motion is

\[ x(t) = \frac{6}{\pi} \sin \left( \frac{\pi}{9} t \right) \]

where \( t \) is in seconds and \( x \) in centimeters. Find the velocity in cm/sec when \( t = 3 \) seconds.

(A) 0
(B) \( \frac{1}{3} \)
(C) \(-1/(4\pi)\)
(D) \( \frac{1}{6} \)
(E) \(-3/(4\pi)\)
(F) \( \frac{5}{6} \)
(G) \(-2/(3\pi)\)
(H) 1
(I) \(-4/(3\pi)\)
(J) 2

\[ \text{vel.} = x' = \frac{6}{\pi} \cdot \frac{\pi}{9} \cos \left( \frac{\pi}{9} t \right) \quad \text{(use chain rule)} \]

\[ \text{vel} = \frac{2}{3} \cos \left( \frac{\pi}{9} t \right) \]

\[ \text{vel}(3) = \frac{2}{3} \cos \left( \frac{\pi}{3} \right) = \frac{2}{3} \cos \left( \frac{\pi}{3} \right) = \frac{1}{3} \frac{\text{cm}}{\text{sec}}. \]
(11) If \( g(x) + x \sin(g(x)) = x^2 + 3x + \frac{\pi}{2} \), find \( g'(0) \).

\[
\begin{align*}
(A) & \quad 0 \\
(B) & \quad 1/2 \\
(C) & \quad \pi/2 \\
(D) & \quad 1 \\
(E) & \quad \pi/3 \\
(F) & \quad 3/2 \\
(G) & \quad 2 \\
(H) & \quad \pi/4 \\
(I) & \quad 5/2 \\
(J) & \quad \pi 
\end{align*}
\]

\[
\begin{align*}
g'(x) + x \cos(g(x)) \cdot g'(x) &= 2x + 3 \\
g'(x)(1 + x \cos(g(x))) &= 2x + 3 - x \sin(g(x)) \\
g'(x) &= \frac{2x + 3 - x \sin(g(x))}{1 + x \cos(g(x))} \\
\end{align*}
\]

\[
\begin{align*}
\text{At } x = 0, \quad g(0) + 0 &= 0 + 0 + \frac{\pi}{2} \implies g(0) = \frac{\pi}{2}. \\
\sin(g(0)) &= \sin\left(\frac{\pi}{2}\right) = 1 \\
\cos(g(0)) &= \cos\left(\frac{\pi}{2}\right) = 0 \\
\end{align*}
\]

\[
\begin{align*}
g'(0) &= \frac{0 + 3 - 1}{1 + 0} = 2
\end{align*}
\]
(12) Find \( \frac{dy}{dx} \) for \( y = \arctan \sqrt{x} \).

\[
\frac{dy}{dx} = \frac{1}{1 + (\sqrt{x})^2} \quad (\sqrt{x})' = \frac{1}{2\sqrt{x}}
\]

(A) \( \frac{1}{1 + x} \)

(B) \( \frac{1}{1 + x + 2\sqrt{x}} \)

(C) \( \frac{1}{1 + 2\sqrt{x}} \)

(D) \( \frac{1}{\sqrt{1-x}} \)

(E) \( \frac{1}{\sqrt{1-\frac{1}{x}}} \)

(F) \( \frac{1}{\sqrt{1-x^2}} \)

(G) \( \sec^2 \sqrt{x} \)

(H) \( \frac{\sec^2 \sqrt{x}}{2\sqrt{x}} \)

(I) \( \frac{\sec \sqrt{x} \tan \sqrt{x}}{2\sqrt{x}} \)

(J) \( \frac{1}{1 + 2x} \)
(13) For $f(x) = \ln |\cos x|$, find $f'(x)$.

(A) $\cot x$

(B) $-\tan x$

(C) $\sec x$

(D) $-\cos x$

(E) $-\sin x$

(F) $-\ln |\cos x| \sin x$

(G) $\ln |\sin x|$

(H) $-\cos x \sin x$

(I) $\sin 2x$

(J) Doesn't exist.

\[
\begin{align*}
\frac{d}{dx} \ln |\cos x| &= \frac{-\sin x}{\cos x} = -\tan x
\end{align*}
\]
(14) Find the linear approximation to \( f(x) = \sqrt{1-x} \) at \( x = 0 \).

(A) \( y = 1 \)
(B) \( y = -\frac{1}{3}x \)
(C) \( y = \frac{1}{3} - x \)
(D) \( y = 3x \)
(E) \( y = -3x \)
(F) \( y = 1 - \frac{1}{3}x \)
(G) \( y = 1 + \frac{2}{3}x \)
(H) \( y = 1 + 3x \)
(I) \( y = x - \frac{2}{3} \)
(J) \( y = 0 \)

\[
y = f(0) + f'(0) (x-0)
\]

\[
f(0) = \frac{3}{\sqrt{1}} = 1
\]

\[
f'(x) = \left( (1-x)^{\frac{1}{2}} \right)' = \frac{1}{2} (1-x)^{-\frac{1}{2}} (-1) \quad \text{(chain rule)}
\]

\[
f'(0) = -\frac{1}{2}
\]

Lin approx is \( y = 1 - \frac{1}{3}x \)
(15) Find the points \((x, y)\) on the curve \(y = 2x^3 + 3x^2 - 12x + 1\) where the tangent line is horizontal. (Caution: be sure to find both \(x\) and \(y\) coordinates for the points asked for).

\[
\text{Tangent line is horizontal when } \frac{dy}{dx} = 0.
\]

\[
\frac{dy}{dx} = 6x^2 + 6x - 12 = 6(x^2 + x - 2) = 6(x+2)(x-1)
\]

is 0 when \(x = -2\) or \(x = 1\)

\(x = -2:\) \quad y(-2) = 2 \cdot (-2)^3 + 3 \cdot (-2)^2 - 12 \cdot (-2) + 1 = -16 + 12 + 24 + 1 = 21

Tangent line is horizontal at \((-2, 21)\).

\(x = 1:\) \quad y(1) = 2 + 3 - 12 + 1 = -6

Tangent line at \((1, -6)\) is horizontal.
(16) Find the derivative of \( f(x) = \sqrt[4]{1 + 2x + x^3} \).

\[
\frac{d}{dx} \left( \sqrt[4]{1 + 2x + x^3} \right) = \frac{1}{4} \left( 1 + 2x + x^3 \right)^{-3/4} \left( 2 + 3x^2 \right)
\]
(17) A curve $C$ is defined by the parametric equations
\[ x = t^2, \quad y = t^3 - 3t \]

It has two tangent lines at the point $(3,0)$. Find their equations.

Slope of the tangent line is
\[ \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2 - 3}{2t} \]

At $(3,0)$ we have $x = 3$, so $t^2 = 3$, so $t = \pm \sqrt{3}$
and then $y = t(t^2 - 3) = 0$ automatically.

The curve passes through $(3,0)$ at $t = -\sqrt{3}$ and at $t = +\sqrt{3}$.

When $t = -\sqrt{3}$,
\[ \frac{dy}{dx} = \frac{3(-\sqrt{3})^2 - 3}{-2\sqrt{3}} = \frac{9 - 3}{-2\sqrt{3}} = \frac{6}{-2\sqrt{3}} = -\sqrt{3} \]
so the corresponding tangent line is
\[ y = -\sqrt{3}(x-3) \]

When $t = \sqrt{3}$,
\[ \frac{dy}{dx} = \frac{3(\sqrt{3})^2 - 3}{2\sqrt{3}} = \frac{9 - 3}{2\sqrt{3}} = \frac{6}{2\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3} \]
so the corresponding tangent line is
\[ y = \sqrt{3}(x-3) \].