Math 131, Fall 2008 - Exam 3

NAME:

STUDENT ID NUMBER:

This exam contains fourteen questions. The first twelve are multiple choice questions and count for five points each. There is no partial credit on these questions, so **read each question carefully**, check your arithmetic and make sure that you have marked the answer you intended to mark. The last two questions, which are each worth twenty points, require written answers, and some partial credit might be given. However, no credit will be given for information that is not germane to the problem at hand. Please make sure to write your name and student ID number on the pages that include your answers to the last two questions. In fact, **you will get one point on each of these two questions for writing your name and ID number legibly.** Also, **please make it clear on your written answer pages what is your actual answer**. If you write something that ends up not being part of your answer, *cross it out*!

1. What is the slope of the tangent line to $y = \sin^{-1} x$ at $x = 0$?

   (a) $-\sqrt{2}/2$.
   (b) $-1/2$.
   (c) -1.
   (d) 0.
   (e) $1/2$.
   (f) 1.
   (g) $\sqrt{2}/2$.
   (h) $\sqrt{3}/2$.

2. If we use linearization to estimate $\sqrt{4.1}$, using $a = 4$, what will our estimate be?

   (a) 5/2.
   (b) 79/40.
   (c) 81/40.
   (d) 21/20.
   (e) 2.
   (f) 19/20.
   (g) 1/20.
   (h) 0.

3. What are the minimum and maximum values attained by $f(x) = x^3 + 3x^2 + 3x + 2$ on the interval $[-2, 1]$?

   (a) $0, 1$.
   (b) $0, 9$.
   (c) $1, 9$.
   (d) $-2, 0$.
   (e) $-2, 1$.
   (f) $0, 1$.
   (g) $1, 2$.
   (h) $-1, 2$.

4. If the graph of a function $y = f(x)$ is a piece of the curve described by the parametric equations $x = t^2$, $y = t^3 + t$, what is $\frac{dy}{dx}$?

   (a) $\frac{t^3+t}{t^2}$.
   (b) $\frac{t^2}{t^3+t}$.
   (c) $3x^2 + 1$.
   (d) $2x$.
   (e) 0.
   (f) $\frac{3t^2+1}{2t}$.
   (g) $\frac{y}{x}$.
   (h) $\frac{2t}{3t^2+1}$.

5. If I charge an admission fee of $\frac{10}{e^x}$ dollars for my lecture, x students will attend the lecture (allowing for a noninteger number of students, for some reason). What adminssion fee, in dollars, should I charge in order to maximize the amount of money I take in?

   (a) $10/e$.
   (b) $e+1$.
   (c) $e-1$.
   (d) 10.
   (e) $-e$.
   (f) e.
   (g) -10.
   (h) 10.

6. What is the largest area, in square feet, that I can fence in with a rectangular fence using exactly 200 feet of fence material?

   (a) $40,000$.
   (b) 800.
   (c) $2,500$.
   (d) 0.
   (e) 200.
   (f) 400.
   (g) $10,000$.
   (h) $4,000$.

7. Which of the following statements are true?

   (I) Every function f that satisfies $f'(x) > 0$ for all x in some interval (a, b) is increasing on (a, b).

   (II) Every function f that satisfies $f'(x) < 0$ for all x in (a, b) is decreasing on (a, b).

   (III) Every function f that satisfies $f'(x) = 0$ for at least one x in (a, b) is neither increasing nor decreasing on (a, b).

   (a) All of them.

   (b) (I) and (II) only.

   (c) (I) and (III) only.

   (d) (II) and (III) only.

   (e) (I) only.

   (f) (II) only.

   (g) (III) only.

   (h) None of them.

8. Which of the following functions have at least one inflection point?

- $f(x) = x^3$.
- $g(x) = x^4$.
- $h(x) = x^4 + 100x$.

(a) All of them.

(b) f and g only.

(c) f and h only.

(d) g and h only.

(e) f only.

(f) g only.

(g) h only.

(h) None of them.

9. What are the absolute minimum and absolute maximum values attained by $f(x) = e^{\sin x}$ on the interval $[0, \pi]$?

   (a) $0, 1$.
   (b) $1/e, e$.
   (c) $-1, 1$.
   (d) $1, e$.
   (e) $-e, e$.
   (f) $1/e, 1$.
   (g) $-1/e, 1/e$.
   (h) $0, e$.

10. How many functions satisfy $f'(x) = e^x$ for all real numbers x?

(a) None.
(b) One.
(c) Two.
(d) Three.
(e) Five.
(f) Eight.
(g) Thirteen.
(h) Infinitely many.

11. How many functions f satisfy both $f'(x) = e^x$ for all real numbers x and $f(11) = 57.66$?

   (a) None.
   (b) One.
   (c) Two.
   (d) Three.
   (e) Five.
   (f) Eight.
   (g) Thirteen.
   (h) Infinitely many.

12. Which of the following statements are true for every function f such that $f''(x)$ is continuous on some open interval containing c?

(I) If $f'(c) = 0$ and $f''(c) > 0$ then f has a local minimum at c.

(II) If $f'(c) = 0$ and $f''(c) < 0$ then f has a local maximum at c.

(III) If $f'(c) = 0$ and $f''(c) = 0$ then f has neither a local maximum nor a local minimum at c.

(a) All of them.
(b) (I) and (II) only.
(c) (I) and (III) only.
(d) (II) and (III) only.
(e) (I) only.
(f) (II) only.
(g) (III) only.
(h) None of them.

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13. Say $f(x) = x^4 e^x$. Find all critical points of f. On which intervals is f increasing? On which intervals is it decreasing? Find all inflection points of f. On which intervals is f concave up? On which intervals is f concave down?

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14. State Rolle's Theorem and the Mean Value Theorem. Give an example that shows that the conclusion of Rolle's Theorem need not hold if we don't assume that f is differentiable on $[a, b]$. If you like, you may use a graph rather than an algebraic description of your function, but in any case you must explain clearly why your function does not satisfy the conclusion of Rolle's theorem.

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