EXAM 3, MATH 131
WEDNESDAY, APRIL 5, 2006

Problems 1 through 14 are multiple choice and worth five points apiece. Problems 15 through 17 are hand-graded and worth ten points apiece. There is a total of 100 points for the whole examination. Do all work in the exam booklet. You may use a scientific calculator, but NOT a graphing calculator.

1) Find the differential of \( y = x^2 \sin 2x \).

(A) \((2x + 2 \cos 2x) \, dx\)
(B) \(2x \cos 2x \, dx\)
(C) \(4x \cos 2x \, dx\)
(D) \(2x^2 \cos 2x \, dx\)
(E) \((2x \sin 2x - 2x^2 \cos 2x) \, dx\)
(F) \(2(x - x^2) \sin 2x \, dx\)
(G) \((2x \sin 2x + 2x^2 \cos 2x) \, dx\)
(H) \(2x \sin 2x + 2x^2 \sec^2 x \, dx\)
(I) \(x \sin 2x + x^2 \cos 2x \, dx\)
(J) \(x^2 \sin 2x \, dx\)
(2) The edge of a cube was measured to be 2 meters with a possible error of 1 centimeter. Find the estimate in $m^3$ obtained by differentials of the maximum possible error in the calculated volume.

(A) .12
(B) .13
(C) .14
(D) .15
(E) .16
(F) .17
(G) .18
(H) .19
(I) .20
(J) .21
(3) If \( x \) and \( y \) are functions of \( t \) such that for every \( t \) they satisfy \( y = x^3 + 2x \), and if \( \frac{dx}{dt} = 5 \), find \( \frac{dy}{dt} \) when \( x = 2 \).

(A) 66
(B) 67
(C) 68
(D) 69
(E) 70
(F) 71
(G) 72
(H) 73
(I) 74
(j) 75
(4) Each side of a square is increasing at a rate of 6 cm/s. Find the rate of increase in the area of the square (in cm²/s) when the area of the square is 25 cm².

(A) 53
(B) 54
(C) 55
(D) 56
(E) 57
(F) 58
(G) 59
(H) 60
(I) 61
(J) 62
(5) A television camera is positioned 3000 ft from the base of a rocket launching pad. The angle of elevation of the camera has to change at the correct rate in order to keep the rocket in sight. The rocket rises vertically. At the moment when the rocket has risen 4000 ft, its speed is 500 ft/s. If the television camera is always kept aimed at the rocket, find the rate of change of the camera’s angle of elevation (in radians/s) at this moment.

(A) .01
(B) .02
(C) .03
(D) .04
(E) .05
(F) .06
(G) .07
(H) .08
(I) .09
(J) .1
(6) A street light is mounted at the top of a 15-ft-tall pole. A man 6 ft tall walks away from the pole with a speed of 6 ft/s along a straight path. Find the speed his shadow is moving in ft/s when he is 30 ft from the pole.

(A) 7
(B) 8
(C) 9
(D) 10
(E) 11
(F) 12
(G) 13
(H) 14
(I) 15
(J) 16
(7) Find the critical numbers of \( f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 6x + 2 \).

(A) 3 and 2
(B) -3 and -2
(C) -3 and 2
(D) 3 and -2
(E) 1 and 6
(F) -1 and -6
(G) -1 and 6
(H) 1 and -6
(I) 2 and 1
(J) -2 and -1
Find the critical numbers of the function on the interval \([-3, 2]\) whose graph is

(A) \(-2.5, -2, -1.5, -0.5, 0\)
(B) \(-1, -0.5, 0, 0.5, 1\)
(C) \(-1.5, 0, 0.5, 1, 2\)
(D) \(-1, 0, 0.5, 1, 1.5\)
(E) \(-2, -1, 0, 1, 2\)
(F) \(-1.5, -1, -0.5, 0.5, 1\)
(G) \(-2, -0.5, 0.5, 1, 1.5\)
(H) \(-1.5, -0.5, 0, 0.5, 1.5\)
(I) \(-1, -0.5, 0, 0.5, 1.5\)
(J) \(-2, -0.5, 0, 1, 1.5\)
(9) Find where the graph of \( f(x) = 2x^3 - 3x^2 - 12x + 9 \) is concave up.

(A) \( \left( \frac{1}{2}, 2 \right) \) only.
(B) \( (-1, \frac{5}{3}) \) only.
(C) Nowhere
(D) \( (2, \infty) \) only.
(E) Everywhere
(F) \( (-\infty, -1) \) and \( (2, \infty) \) only.
(G) \( (-1, 2) \) only.
(H) \( (-\infty, -1) \) only.
(I) \( \left( \frac{1}{2}, \infty \right) \) only.
(J) \( (-\infty, \frac{1}{2}) \) only.
Consider the function \( f(x) = x - 2 \sin x \) on the interval \( 0 \leq x \leq 3\pi \). A sign diagram of its derivative \( f' \) is

\[
\begin{array}{ccccccccccc}
\text{Interval} & 0 & \frac{\pi}{3} & \frac{2\pi}{3} & \pi & \frac{4\pi}{3} & \frac{5\pi}{3} & 2\pi & \frac{7\pi}{3} & \frac{8\pi}{3} & 3\pi \\
\text{Sign of } f' & - & + & + & + & + & + & - & - & - & + \\
\end{array}
\]

Find the number at which the absolute minimum of \( f \) occurs.

(A) 0
(B) 3\pi
(C) 8\pi/3
(D) \pi/3
(E) 2\pi/3
(F) 7\pi/3
(G) 2\pi
(H) \pi
(I) 4\pi/3
(J) 5\pi/3
(11) Find the absolute \textbf{maximum value} of \( f(x) = x\sqrt{4-x^2} \) on the interval \([0, 2]\).

(A) 0
(B) 1/2
(C) 2/3
(D) 1
(E) \sqrt{3}
(F) 2
(G) \sqrt{5}
(H) 2\sqrt{3}
(I) 5/2
(J) 3
(12) If $f$ is a differentiable function on the interval $[2, 8]$ and if $f(2) = 4$ and $f(8) = 6$, then the Mean Value Theorem says there is some number $c$ between 2 and 8 such that $f'(c)$ is what? Find $f''(c)$.

(A) 0
(B) 8
(C) 1
(D) 1/2
(E) 1/3
(F) 1/5
(G) 1/6
(H) 1/8
(I) 24
(J) $\infty$
(13) Find
\[
\lim_{x \to 0} \frac{6^x - 2^x}{x}
\]

(A) \infty
(B) 0
(C) 2
(D) 3
(E) 4
(F) \ln 6
(G) \ln 5
(H) \ln 4
(I) \ln 3
(J) \ln 2
(14) Find

\[ \lim_{x \to 0} \frac{e^{3x} - 1 - 3x}{x^2} \]

(A) 0  
(B) \infty  
(C) \frac{3}{4}  
(D) 1  
(E) 2  
(F) \frac{5}{2}  
(G) 3  
(H) \frac{7}{2}  
(I) 4  
(J) \frac{9}{2}
The next three problems are hand graded. Show your work and clearly indicate your answer.

(15) A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 2 ft/s, how fast is the top of the ladder sliding down the wall when the bottom is 8 ft from the wall? (Partial credit for a correct diagram with the variables identified on it).
(16) Find the absolute maximum and the absolute minimum of \( f(x) = 3x^2 - 12x + 5 \) on the interval \([0, 3]\).
(17) Find

\[ \lim_{x \to 0} (1 + 2x)^{1/x} \]