MATH 131 **EXAM III** F05M131.1.1

NAME : Solutions ID NUMBER_____

PART I consists of 12 multiple choice questions (worth 5 points each) for a total of 60 points. Mark the correct answer on the answer card . For part I only the answer on the card will be graded.
Part II consists of 4 handgraded problems (worth 10 points each) for a total of 40 points . A correct answer without supporting work may get no credit . Present a readable orderly sequence of steps showing how you got your answer .

Part I (60 points) :

1) Find $\lim_{x\to 1} \frac{ln(x^2)}{x-1}$. $\left(\frac{0}{0}\right)$

A) 0
B) 1
C) 2
D) $\frac{1}{2}$
E) 3
F) $\frac{1}{3}$
G) $\frac{2}{3}$
H) 4
I) $\frac{3}{4}$
J) DNE

$$= \lim_{x\to 1} \frac{\frac{2x}{x^2}}{1} = \lim_{x\to 1} \frac{2}{x} = 2$$

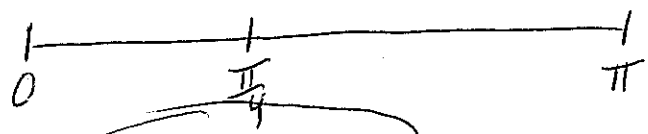
2) Find the **absolute maximum** (y-coordinate) of the curve $y = e^x \cos(x)$ on the closed interval $[0, \pi]$.

A) 1.45
B) 1.50
C) 1.55
D) 1.60
E) 1.65
F) 1.70
G) 1.75
H) 1.80
I) 1.85
J) 1.90

$$\frac{dy}{dx} = e^x \cos(x) - e^x \sin(x)$$
$$= e^x(\cos(x) - \sin(x)) = 0$$

$\sin(x) = \cos(x)$ only at $\frac{\pi}{4}$

for $0 \le x \le \pi$



$f(0) = 1$ $f(\frac{\pi}{4}) \sim 1.55$ $f(\pi) = -e^{\pi}$

3) Find ***all*** the critical values (x-coordinates) of the function $f(x) = \sqrt{x^2 - 4x}$.

A) $-1, 0, 1$
B) $0, 1$
C) $-1, 1, 2$
D) $0, 1, 2$
E) $0, 2$
F) $1, 2, 3$
G) $2, 3$
H) $2, 4$
I) $0, 2, 4$
J) $0, 4$

$x^2 - 4x = x(x-4)$ so

domain of $f(x)$ is $x \le 0$ or $x \ge 4$

$f'(x) = \frac{1}{2}(x^2-4x)^{-1/2}(2x-4)$

$= \dfrac{x-2}{\sqrt{x^2-4x}}$

$x=2$ is not in domain

hence critical values are only where f' DNE and that is at $\boxed{x=0,\ x=4}$

4) On which of the following intervals is $f(x) = \sqrt{9 - x^2}$ *increasing* ?

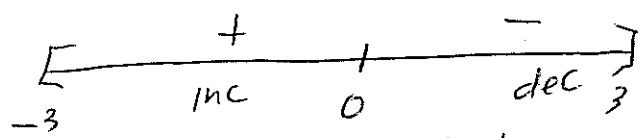
A) $(0, 3)$
B) $(-1, 1)$
C) $(1, 3)$
D) $(-2, 2)$
E) $(0, 2)$
F) $(-2, 0)$
G) $(-1, 3)$
H) $(2, 4)$
I) $(-4, 0)$
J) $(-1, 4)$

Domain
$-3 \le x \le 3$

$f'(x) = \frac{1}{2}(9-x^2)^{-\frac{1}{2}}(-2x)$

$= \dfrac{-x}{\sqrt{9-x^2}} = 0$

$x = 0$



The only interval listed on which $f(x)$ is increasing is $(-2, 0)$. All the others contain points at which $f(x)$ is decreasing.

5) Find $F(x) = \int \frac{2}{\sqrt{x}}\, dx$, the **antiderivative** of $f(x) = \frac{2}{\sqrt{x}}$.

A) $2x^{-3/2} + C$
B) $-2x^{-3/2} + C$
C) $x^{-3/2} + C$
D) $-x^{-3/2} + C$
E) $x^{1/2} + C$
F) $-x^{1/2} + C$
G) $2x^{1/2} + C$
H) $-2x^{1/2} + C$
I) $4x^{1/2} + C$
J) $-4x^{1/2} + C$

$F(x) = 2X^{-\frac{1}{2}}$

so $F(x) = 4X^{\frac{1}{2}} + C$

6) Find $\lim_{x\to\infty} \frac{ln(8x^3)}{x}$: $\left(\frac{\infty}{\infty}\right)$

A) 0

B) ∞

C) $-\infty$

D) 24

E) $\frac{1}{3}$

F) 8

G) $\frac{1}{8}$

H) $\frac{3}{8}$

I) $\frac{8}{3}$

$= \lim_{x\to\infty} \frac{\frac{24X^2}{8X^3}}{1} =$

$\lim_{x\to\infty} \frac{3}{X} = 0$

7) Find $\lim_{x \to 0^+} (1-x)^{1/x}$ (1^∞)

A) 0
B) 1
C) e
D) $\frac{1}{e}$
E) $-e$
F) $-\frac{1}{e}$
G) ∞
H) $-\infty$
I) e^2
J) e^{-2}

Take $\lim_{x \to 0^+} \ln\left((1-x)^{1/x}\right)$

$= \lim_{x \to 0^+} \frac{\ln(1-x)}{x}$ $\left(\frac{0}{0}\right)$

$= \lim_{x \to 0^+} \frac{\frac{-1}{1-x}}{1} = -1$

then our problem

$\lim_{x \to 0^+} (1-x)^{1/x} = e^{-1} = \frac{1}{e}$

8) For the curve $y = 2x^5 - 5x^4 + 3x - 10$, which of the following is the x-coordinate of a **point** of **inflection**.

A) 0
B) $\frac{1}{2}$
C) 1
D) $\frac{3}{2}$
E) 2
F) $-\frac{1}{2}$
G) -1
H) $-\frac{3}{2}$
I) -2
J) no point of inflection

$y' = 10x^4 - 20x^3 + 3$

$y'' = 40x^3 - 60x^2$

$= 20x^2(2x-3) = 0$

$x = 0, \quad x = \frac{3}{2}$

$-$ $-$ $+$

down 0 down $\frac{3}{2}$ up

$x = \frac{3}{2}$ is only point of inflection

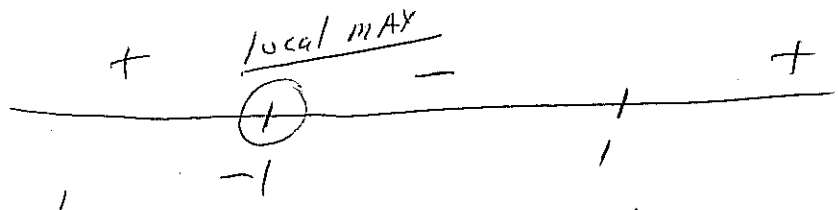
9) Find the x-coordinate of a **local maximun** of $f(x) = x - 3\,x^{1/3}$.

A) 0
B) $\frac{1}{2}$
C) 1
D) $\frac{3}{2}$
E) 2
F) $-\frac{1}{2}$
G) -1
H) $-\frac{3}{2}$
I) -2
J) no local maximum

$$f'(x) = 1 - x^{-\frac{2}{3}}$$

$$= 1 - \frac{1}{x^{2/3}}$$

$$= \frac{x^{2/3} - 1}{x^{2/3}} = 0 \qquad x = \pm 1$$



only $\underline{x = -1}$ is $\underline{\text{local MAX}}$

10) Find $\lim\limits_{x \to 2} \frac{\sqrt{x^2+5} - 3}{x^2 - 4}$ $\left(\frac{0}{0}\right)$

A) 0
B) 1
C) $\frac{1}{2}$
D) 2
E) $\frac{1}{3}$
F) $\frac{1}{4}$
G) $\frac{1}{5}$
H) $\frac{1}{6}$
I) $\frac{1}{8}$
J) ∞

$$= \lim_{x \to 2} \frac{\frac{1}{2}(x^2+5)^{-1/2} \cdot \cancel{2x}}{\cancel{2x}}$$

$$= \lim_{x \to 2} \frac{1}{2\sqrt{x^2+5}} = \boxed{\frac{1}{6}}$$

11) Among all pairs of positive numbers whose product is 36 , find the **smallest number** that their **sum** can be .

A) 2
B) 4
C) 6
D) 8
E) 10
F) 12
G) 14
H) 16
I) 18
J) 20

Let x, y be the two pos. number

Given $x \cdot y = 36$ or $\boxed{y = \frac{36}{x}}$

Find absolute Minimum of

sum , $S = x + y = x + \frac{36}{x}$

$\frac{dS}{dx} = 1 - \frac{36}{x^2} = 0$ Only if $\boxed{x = 6}$

Then $\boxed{y = 6}$ and

$\boxed{\text{sum} = 12}$

12) Find all the **vertical asymptotes** of $f(x) = \frac{x^2 - 5x + 6}{3x^2 + 4x + 2}$

A) $x = 2, x = 3$
B) $x = 1, x = 3$
C) $x = -1, x = -\frac{1}{3}$
D) $x = 1, x = \frac{2}{3}$
E) $x = \frac{2}{3}, x = 3$
F) $x = -\frac{2}{3}, x = 3$
G) $x = \frac{1}{3}, x = 2$
H) $x = -2, x = -\frac{2}{3}$
I) $x = -\frac{1}{3}, x = \frac{1}{3}$
J) no vertical asymptotes

denominator $3x^2 + 4x + 2$ is always positive , never equal to zero .

You can check by part quadratic formula for roots

$b^2 - 4ac = (4)^2 - 4(3)(2) = 16 - 24 < 0$

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PART II : Show the work you did to get the answer in a readable orderly form .

13) Let $f(x) = x^4 - 4x^3 + 10$

a) Where is the function **increasing** and where is it **decreasing** ? (3 pts)

$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3) = 0$

only if $x=0$, $x=3$

$-$ $-$ $+$

0 3

decreasing $-\infty < x < 3$ or $\{-\infty < x < 0\} \cup \{0 < x < 3\}$

increasing $x > 3$

b) Where is the function **concave up** and where is it **concave down** ? (3 pts)

$f''(x) = 12x^2 - 24x = 12x(x-2) = 0$

$x = 0$, $x = 2$

$+$ $-$ $+$

0 2

Concave up

$-\infty < x < 0 \cup x > 2$

Concave down

$0 < x < 2$

c) Label each point (**x & y coordinates**) which is a **local max** , **local min** or an **inflection point** .

local min at $(3, -17)$

no local MAX

inflection pts : $(0, 10)$, $(2, -6)$

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14) The **Surface Area** of a cylinder (including top and bottom) is :
$A = 2\pi r h + 2\pi r^2$ where r = radius and h = height .

a) If the **volume** of a cylinder is 80 cm^3 , find a formula for the **Surface Area** of that cylinder , **A(r)** , written as a function of r alone. (4 pts)

Give volume $= \pi r^2 h = 80$

then $\boxed{h = \frac{80}{\pi r^2}}$

So $A(r) = 2\pi r\left(\frac{80}{\pi r^2}\right) + 2\pi r^2$

$$\boxed{A(r) = \frac{160}{r} + 2\pi r^2}$$

b) Find the **radius** of the cylinder of the type described in part (a) which would have the **smallest possible surface area** . (6 pts)

$$A'(r) = -\frac{160}{r^2} + 4\pi r = \frac{-160 + 4\pi r^3}{r^2} = 0$$

$$4\pi r^3 = 160$$

$$r^3 = \frac{160}{4\pi} = \frac{40}{\pi}$$

$$\boxed{r = \sqrt[3]{\frac{40}{\pi}}}$$

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PART II : Show the work you did to get the answer in a readable orderly form .

15) Consider $\lim_{x\to\infty} \sqrt{x^2+4x} - x$.

a) **What type** of indeterminate form is it ? (2 pts)

$\infty - \infty$

b) Use factoring to rewrite it as **inderminate form** of type $\infty \cdot 0$. (3 *pts*)

$$\lim_{x\to\infty} \sqrt{x^2\left(1+\frac{4}{x}\right)} - x = \lim_{x\to\infty} x\sqrt{1+\frac{4}{x}} - x$$

$$= \boxed{\lim_{x\to\infty} x\left(\sqrt{1+\frac{4}{x}} - 1\right)}$$

c) Use **L'Hospital's Rule** to **solve** this limit . (5 pts)

$$= \lim_{x\to\infty} \frac{\sqrt{1+\frac{4}{x}} - 1}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\frac{-\frac{4}{x^2}}{2\sqrt{1+\frac{4}{x}}}}{-\frac{1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{1}{2\sqrt{1+\frac{4}{x}}} \cdot \left(\cancel{-}\frac{4}{\cancel{x^2}}\right)\left(\cancel{-}\frac{\cancel{x^2}}{1}\right) = 2$$

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16)

a) Find the general formula for $\int (3\sqrt{x} + \frac{5}{x^6})\, dx$. (5 *pts*)

$$= \int 3x^{1/2} + 5x^{-6}\, dx$$

$$= 3 \cdot \frac{2}{3} x^{3/2} + 5 \cdot \frac{x^{-5}}{-5} + C$$

$$= \boxed{2x^{3/2} - \frac{1}{x^5} + C}$$

b) Find f(x) given that $f''(x) = \frac{3}{\sqrt{x}}$, $f'(4) = 7$ and $f(4) = 20$. (5 pts)

$f''(x) = 3x^{-\frac{1}{2}}$ then $f'(x) = 3 \cdot 2x^{1/2} + C$

$f'(x) = 6x^{1/2} + C$ $\quad f'(4) = 6\sqrt{4} + C$

$= 12 + C = 7$

so $\boxed{C = -5}$

$f'(x) = 6x^{1/2} - 5$

Next: $f(x) = 6 \cdot \frac{2}{3} x^{3/2} - 5x + D$

$= 4x^{3/2} - 5x + D$

$f(4) = 4(4)^{3/2} - 5 \cdot 4 + D = 20$

so $32 - 20 + D = 20 \Rightarrow \boxed{D = 8}$

$$\boxed{f(x) = 4x^{3/2} - 5x + 8}$$