This exam should have 15 questions. The first 10 questions are multiple choice questions worth 7 points each. The next three questions are TRUE or FALSE questions worth 4 points each. The last two questions are hand graded and are worth 9 points each. Please check to see that your exam is complete. If you do not have a pencil to mark your card please request your proctor for one.

Write your ID number (not your SS number) on the six blank lines at the top of your answer card using one blank for each digit. Shade in the corresponding boxes below. Also print your name at the top of your card.

As you work the exam, lightly shade in the correct answers on your answer card. At the end of your exam when you are certain of all your choices darken all your answer boxes. If your card becomes damaged please ask your proctor for a new one.

Instructions for the hand graded part

Problems 14 and 15 are hand graded and each is worth 9 points. Write your answers on the test page. Show your work neatly and cross out irrelevant scratch work, false starts etc. A mere final answer will only get partial credit. You must show the complete work showing all steps.

Please put your name on each of the pages containing Problems 14 and 15, since they may be separated during grading. Also please add your Discussion Section Letter, which can be found on the first page of this exam, on both these pages.
1. Which function below satisfies the differential equation $y'' + y = 2\cos x$?

(A) $\sin x$

(B) $2\sin x$

(C) $\cos x$

(D) $2\cos x$

(E) $x\sin x$

(F) $x\cos x$

(G) $2x\sin x$

(H) $2x\cos x$

(I) $\sin x + \cos x$

(J) $\sin x - \cos x$
2. The solution to the initial value problem \( y' = \cos x, \ y(0) = 0 \) also satisfies one of the following differential equations. Identify this differential equation.

(A) \( y' = \sin x \)
(B) \( y' = y \sin x \)
(C) \( y' = y \cos x \)
(D) \( y' = \frac{y}{\sin 2x} \)
(E) \( y' = \frac{y}{\cos 2x} \)
(F) \( y' = \frac{\sin 2x}{2y} \)
(G) \( y' = \frac{\cos 2x}{2y} \)
(H) \( y' = \frac{\sin 2x}{y} \)
(I) \( y' = \frac{\cos 2x}{y} \)
(J) \( y' = \frac{\sin 2x}{\cos 2x} \)
3. Using Euler’s method with step size 0.1 compute \( I(0.3) \) where \( I \) satisfies the initial value problem \( \frac{dI}{dt} = 15 - 3I \) with \( I(0) = 0 \).

(A) 1.5
(B) 2.35
(C) 3.285
(D) 4.614
(E) 5.726
(F) 6.341
(G) 7.912
(H) 8.423
(I) 9.013
(J) 10.239
4. Solve the initial value problem \( \frac{dz}{dt} + e^{z+t} = 0 \), \( z(0) = 0 \).

(A) \( z = \ln(1 - t) \)
(B) \( z = \ln(1 + t) \)
(C) \( z = e^t - 1 \)
(D) \( z = 1 - e^t \)
(E) \( z = e^t - e^{-t} \)
(F) \( z = t \)
(G) \( z = -t \)
(H) \( z = te^t \)
(I) \( z = te^{-t} \)
(J) \( z = t(e^t + e^{-t}) \)
5. Find the orthogonal trajectories of the family of curves \( y = \frac{k}{x} \).

(A) \( x + y = C \)
(B) \( x - y = C \)
(C) \( x^2 + y^2 = C \)
(D) \( x^2 - y^2 = C \)
(E) \( y = C \ln x \)
(F) \( y = e^{Cx} \)
(G) \( y = Ce^x \)
(H) \( y = \ln Cx \)
(I) \( xy = C \)
(J) \( y = Cx \)
6. When a cold drink is taken out from the refrigerator, its temperature is $5^\circ C$. The room temperature is $20^\circ C$. After 25 minutes the temperature of the drink has increased to $10^\circ C$. What is the approximate temperature of the drink after 50 minutes? Choose the number closest to your answer.

(A) $11.3^\circ C$
(B) $11.7^\circ C$
(C) $12.3^\circ C$
(D) $12.7^\circ C$
(E) $13.3^\circ C$
(F) $13.7^\circ C$
(G) $14.3^\circ C$
(H) $14.7^\circ C$
(I) $15.3^\circ C$
(J) $15.7^\circ C$
7. If $P$ satisfies the initial value problem $\frac{dP}{dt} = P(1 - \frac{P}{100})$ with $P(0) = 10$, find the value of $t$ so that $P(t) = 50$.

(A) $\ln 2$
(B) $\ln 3$
(C) $\ln 4$
(D) $\ln 5$
(E) $\ln 6$
(F) $\ln 7$
(G) $\ln 8$
(H) $\ln 9$
(I) $\ln 10$
(J) $\ln 11$
8. Find the area of the region enclosed by the curves $y = x^2$ and $y = x^3$.

(A) $\frac{1}{2}$
(B) $\frac{1}{4}$
(C) $\frac{1}{6}$
(D) $\frac{1}{8}$
(E) $\frac{1}{10}$
(F) $\frac{1}{12}$
(G) $\frac{1}{14}$
(H) $\frac{1}{16}$
(I) $\frac{1}{18}$
(J) $\frac{1}{20}$
9. Find the volume of the solid obtained by rotating the region bounded by the curves \( y = 1 - x^2 \) and \( y = 0 \) about the \( x \)-axis.

(A) \( \frac{2\pi}{15} \)
(B) \( \frac{4\pi}{15} \)
(C) \( \frac{2\pi}{3} \)
(D) \( \frac{8\pi}{15} \)
(E) \( \frac{2\pi}{3} \)
(F) \( \frac{4\pi}{3} \)
(G) \( \frac{14\pi}{15} \)
(H) \( \frac{10\pi}{15} \)
(I) \( \frac{6\pi}{5} \)
(J) \( \frac{4\pi}{3} \)
10. Find the arc length of the parametric curve \( x = e^t + e^{-t}, \ y = 5 - 2t, \ 0 \leq t \leq 3 \).

(A) \( e - e^{-1} \)
(B) \( e^2 - e^{-2} \)
(C) \( e^3 - e^{-3} \)
(D) \( e^4 - e^{-4} \)
(E) \( e^5 - e^{-5} \)
(F) \( e^6 - e^{-6} \)
(G) \( e^7 - e^{-7} \)
(H) \( e^8 - e^{-8} \)
(I) \( e^9 - e^{-9} \)
(J) \( e^{10} - e^{-10} \)
These are three TRUE or FALSE questions each worth 4 points.

11. A biologist asked her assistant to grow some bacteria. It was known that the growth was modeled by the logistic equation \( \frac{dp}{dt} = 0.5P(1 - \frac{P}{1000}) \). The initial population was 100. After several days the biologist asked the assistant how many bacteria were found and the assistant replied 'around 2000'. The assistant was speaking

(A) The Truth
(B) a falsehood
12. The area of the region enclosed by the two curves \( y = x^{7036} \) and \( y = x^{7037} \) is given by the formula \( \int_{-1}^{1} (x^{7035} - x^{7037}) \, dx \).

(A) TRUE  
(B) FALSE
13. (Treat this as a formal mixing problem.) Professor Snape took Harry Potter to the Dark Lake which was churning violently. Snape pointed out to Harry that a river of pure water was flowing into the lake and a river was flowing out of the lake at the same rate. Snape said that the all powerful dark wizard Voldemort had sprinkled a few grains of salt into this lake and in a thousand years it will still have salt. Was Snape telling

(A) the TRUTH
(B) a LIE
14. Find the area of the region bounded by the $x$-axis, the $y$-axis, the line $x = 1$ and the parametric curve $x = \cos t, y = t, \ 0 \leq t \leq \pi/2$ by the following procedure:

(A) Make a rough sketch of the region (1 pt)
(B) Set up the integral in terms of $t$ which will compute the area (3 pts)
(C) Evaluate the integral (5 pts)
15. Find an expression for \( y \) in terms of \( x \) explicitly if \( y \) satisfies the initial value problem \( y' = \frac{2y^2 \cos x}{1+y^2} \) with \( y(0) = 1 \) (6 pts). Find all values of \( x \) such that \( y(x) = 1 \) (3 pts).