This exam should have 22 questions. Part I will have 20 multiple choice questions, 4 points each. Part II will have 2 handgraded questions, 10 points each. Please check to see that your exam is complete. If you do not have a PENCIL to mark your card, please ask to borrow one from your proctor.

Write your ID NUMBER (not your SS number) on the six blank lines on the top of your answer card, using one blank for each digit. Shade in the corresponding boxes below. Also Print your name at the top of your card.

**PART I** : (80 points)

1) Find the volume of the solid obtained by rotating the region bounded by the lines $x = 0$, $x = 2$, $y = -4$ and $y = 4$ about the $y$-axis.

A) $\pi$  B) $\frac{10\pi}{3}$  C) $12\pi$  D) $\frac{18\pi}{5}$  E) $24\pi$  F) $\frac{36\pi}{5}$  G) $32\pi$

H) $40\pi$  I) $\frac{46\pi}{7}$  J) $48\pi$
2) Find the volume of the solid obtained by rotating the region bounded by the curve
\[ y = 2\sqrt{x} \] and the line \( y = x \) about the x-axis.

A) \( \pi \)  B) \( \frac{11\pi}{5} \)  C) \( 12\pi \)  D) \( \frac{20\pi}{3} \)  E) \( 24\pi \)  F) \( \frac{32\pi}{3} \)  G) \( 32\pi \)
H) \( 40\pi \)  I) \( \frac{43\pi}{5} \)  J) \( 48\pi \)

3) Find the volume of the solid whose base is the region in the x-y plane that
is bounded by the curve \( x = y^2 \) and the line \( x = 4 \) and whose cross-sections
(slices) perpendicular to the x-axis are squares.

A) 1  B) \( \frac{10}{3} \)  C) 12  D) \( \frac{18}{5} \)  E) 24  F) \( \frac{33}{5} \)  G) 32
H) 40  I) \( \frac{45}{7} \)  J) 48
4) Consider the solid obtained by rotating about the x-axis the region in the first quadrant bounded by the curve \( y = \sqrt{x} \) and the lines \( x = 4 \) and \( y = 0 \). For \( x \) between 0 and 4 write out a formula for \( A(x) \), the cross-section of the solid made by the plane perpendicular to the x-axis at that point.

A) \( x^2 \)  B) \( 2\pi x \)  C) \( \pi x^2 \)  D) \( \pi x \)  E) \( \frac{\pi x}{2} \)  G) \( 2\pi x^2 \)  H) \( \frac{3x^2}{2} \)  I) \( \frac{x}{3} \)  J) \( \frac{2\pi x}{3} \)

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5) Which of the following integrals gives the length of the curve \( y = \frac{4\sqrt{2}}{3} x^{3/2} - 1 \), \( 0 \leq x \leq 1 \).

A) \( \int_{0}^{1} \sqrt{3 + 4x^2} \, dx \)
B) \( \int_{0}^{1} \sqrt{1 + 4x^2} \, dx \)
C) \( \int_{0}^{1} \sqrt{3 + 5x^2} \, dx \)
D) \( \int_{0}^{1} \sqrt{1 + 5x^2} \, dx \)
E) \( \int_{0}^{1} \sqrt{3 + x^2} \, dx \)
F) \( \int_{0}^{1} \sqrt{3 + 8x} \, dx \)
G) \( \int_{0}^{1} \sqrt{1 + 8x} \, dx \)
H) \( \int_{0}^{1} \sqrt{1 + 5x} \, dx \)
I) \( \int_{0}^{1} \sqrt{5 + x} \, dx \)
J) \( \int_{0}^{1} \sqrt{3 + 5x} \, dx \)
6) Find the **average value** of the function \( f(x) = 1 + x^2 \) over \([0, 2]\).

   A) \( \frac{14}{3} \)  B) \( \frac{7}{3} \)  C) 4  D) \( \frac{13}{3} \)  E) \( \frac{14}{3} \)  F) 5  G) \( \frac{4}{3} \)  H) \( \frac{17}{3} \)  I) 6  J) \( \frac{16}{3} \)

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7) Find the **constant**, \( C \), given that the **average value** of the function \( f(x) = x^3 \) over the interval \([0, C]\) is 16.

   A) \( \frac{1}{4} \)  B) \( \frac{1}{2} \)  C) 1  D) \( \frac{3}{2} \)  E) 4  F) \( \frac{4}{3} \)  G) 8  H) \( \frac{8}{3} \)  I) 16  J) \( \frac{16}{3} \)
8) A tank has the shape of a cylinder with radius 2 ft and height 6 ft. Suppose the tank is full of a liquid weighing 80 lb/ft. Find the work needed (in ft-lb) to pump one-third of the liquid from the tank.

A) $120\pi$ B) $230\pi$ C) $345\pi$ D) $460\pi$ E) $520\pi$ F) $640\pi$ G) $730\pi$ H) $840\pi$ I) $890\pi$
9) A spring has a **natural length** of 1 ft. A force of 24 lb stretches the spring to a **length** of 1.8 ft. How much **work** (in ft-lb) will it take to stretch the spring to a **length** of 3 ft?

A) 10  B) 20  C) 30  D) 40  E) 50  F) 60  G) 70  H) 80  I) 90  J) 100

10) It takes 1800 ft-lb of **work** to stretch a spring from its **natural length** of 2 ft to a **length** of 5 ft. How many feet beyond its **natural length** will a force of 2000 lb stretch the spring?

A) 1  B) 2  C) 3  D) 4  E) 5  F) 6  G) 7  H) 8  I) 9  J) 10
11) A motor at the top of an elevator shaft is about to lift a uniform cable 180 ft long weighing 40 lb to the top of the shaft. How much work (in ft-lb) does the motor do in lifting the entire cable to the top of the shaft?

A) 900  B) 1800  C) 2400  D) 3600  E) 4000  F) 4200  
G) 4600  H) 4800  I) 5200  J) 5600

12) Suppose the length of telephone calls from a specific telephone booth is related to an exponential decreasing probability density function with a mean of 4 min. For what number of minutes C is there a 50% probability that a person will speak for at least C minutes? (Answer is to two decimal places)

A) 2.50  B) 2.77  C) 2.93  D) 3.42  E) 3.78  F) 4.00  G) 4.27  H) 4.73  I) 5.14
13) Suppose the shelf-life (in years) of a certain brand of flashlight batteries is a continuous random variable with probability density function

\[ f(x) = \frac{1}{(x+1)^2} \]

What is the probability that such a battery will have a shelf-life less than 3 yrs?

A) 34%  B) 38%  C) 42%  D) 48%  E) 52%  F) 59%  G) 62%  H) 68%  I) 71%  J) 75%

14) For the normal distribution that is based on the probability density function

\[ f(x) = \begin{cases} \frac{1}{3\sqrt{2\pi}} e^{-\frac{(x-4)^2}{3\sigma^2}} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases} \]

what is the mean?

A) 2  B) 3  C) 4  D) 8  E) 10  F) \sqrt{2}  G) \sqrt{2\pi}  H) 9  I) 18  J) 20
15) Find the value of the constant $C$ that would make $y = C - e^{-\frac{x}{2}}$ a solution to the differential equation $y' = 10 - \frac{1}{2} y$.

A) 0  B) 4  C) 8  D) 10  E) 14  F) 16  G) 20  H) 24  I) 28  J) 30

16) If we sketched the direction field of the differential equation $y' = 2y - 3x$, find a line in the x-y plane on which all the line segments crossing that line would have 6 as their slope.

A) $y = 6x + 1$  B) $y = 3x + 6$  C) $y = \frac{1}{6}x + 3$  D) $y = \frac{3}{2}x + 3$  E) $y = \frac{3}{2}x + 6$

F) $y = -\frac{1}{3}x + 4$  G) $y = -\frac{1}{2}x + 4$  H) $y = -\frac{2}{3}x + 6$  I) $y = x$  J) $y = -x$
17) Use Euler's method with step size \( h = 0.2 \) to estimate the value of \( y(1.4) \) where \( y(x) \) is a solution to the differential equation \( y' = y + xy \) , with initial value \( y(1) = 1 \).

A) 1.984  B) 2.016  C) 2.142  D) 2.244  E) 2.322  F) 2.414  G) 2.508  H) 2.626  I) 2.742  J) 2.898

18) Solve the differential equation \( y' = (1 + y^2) \cdot x^2 \), \(-1 < x < 1\), with initial value \( y(0) = 1 \).

A) \( y = \frac{1}{3}x^3 + 1 \)

B) \( y = \frac{2}{3}x^3 + \frac{1}{3} \)

C) \( y = \sin\left(\frac{x^2}{3} + \frac{x}{2}\right) \)

D) \( y = \cos\left(\frac{2x^2}{3} + \frac{x}{6}\right) \)

E) \( y = \tan\left(\frac{x^2}{3} + \frac{x}{4}\right) \)

F) \( y = \sec\left(\frac{2x^3}{3} + \frac{x}{12}\right) \)

G) \( y = \tan\left(\frac{x^3}{3} + \frac{x}{6}\right) \)

H) \( y = \sin\left(\frac{x^2}{3} + \frac{x}{4}\right) \)

I) \( y = \tan\left(\frac{x^2}{3} + \frac{x}{4}\right) \)

J) \( y = \tan\left(\frac{2x^3}{3} + \frac{x}{6}\right) \)
19) In the first quadrant \( x > 0, y > 0 \) which one of the curves below is a member of the orthogonal trajectories of the family of curves \( 2x^2 + y^2 = C \)?

A) \( y = 2x^2 \)

B) \( x^2 - 2y^2 = 1 \)

C) \( y = \frac{2}{3}x^{3/2} \)

D) \( y = x^2 - 2x \)

E) \( x^2 - 4y^2 = 2 \)

F) \( y = (2x)^{1/2} \)

G) \( 2y^2 - x = 4 \)

H) \( y = (x^3 + 1)^{1/2} \)

I) \( x^2 - y^2 = 4 \)

J) \( y = (\frac{1}{3}x + 1)^{3/2} \)
20) A company has a 1000 gal tank which is used to control the release of pollutants into a sewage system. Initially the tank had 500 gal with 1000 lb of pollutants. Then additional polluted water containing 5 lb/gal of pollutants is pumped into the tank at the rate of 100 gal/hr and the liquid is always being mixed thoroughly. At the same time 50 gal/hr (different rate) of the liquid is released into the sewage system. If $y(t)$ is the number of pounds of pollutant in the tank $t$ hours after the start, then find the differential equation satisfied by $y(t)$.

(Note: Since liquid enters at 100 gal/hr and leaves at 50 gal/hr, after every hour we have an additional 50 gal of liquid in the tank.)

A) $y' = 500 - \frac{y}{10 + t}$

B) $y' = 1000 - \frac{y}{10 + t}$

C) $y' = 500 - \frac{50y}{50 + t}$

D) $y' = 1000 - \frac{50y}{10 + t}$

E) $y' = 500 - \frac{y}{50}$

F) $y' = 1000 - \frac{y}{50}$

G) $y' = 500 - \frac{y + 10}{50 + t}$

H) $y' = 1000 - \frac{y + 10}{50 + t}$

I) $y' = 500 - \frac{50y}{t}$

J) $y' = 1000 - \frac{y}{50 + t}$
21) Consider the region $R$ which is bounded by the curve $y = \sqrt{x}$ and the lines $x + y = 2$ and $x = 0$. (Decimal answers will not be accepted.)

a) Find the volume of the solid obtained by rotating $R$ about the $x$-axis (5 pts)

b) Find the volume of the solid obtained by rotating $R$ about the $y$-axis (5 pts)
(Hint: cylindrical shell method is a bit easier)
PART II: CLEARLY WRITE YOUR SOLUTION AND HOW YOU GOT IT

Name: ____________________________

ID #: ____________________________

22) Consider the differential equation \( t + 2y \sqrt{t^2 + 1} \frac{dy}{dt} = 0 , \ y > 0 \).

a) Find the formulas for \( g(t) \) and \( h(y) \) if the differential equation is rewritten in the separable form \( \frac{dy}{dt} = g(t) \cdot h(y) \). (2 pts)

b) Find the general solution for the above differential equation. (5 pts)

c) Find a precise formula for \( y \) if we know that the curve passes through \( (\sqrt{3}, \sqrt{3}) \). (3 pts)