This exam should have 15 questions. The first 10 questions are multiple choice questions worth 7 points each. The next three questions are TRUE or FALSE questions worth 4 points each. The last two questions are hand graded and are worth 9 points each. Please check to see that your exam is complete. If you do not have a pencil to mark your card please request your proctor for one.

Write your ID number (not your SS number) on the six blank lines at the top of your answer card using one blank for each digit. Shade in the corresponding boxes below. Also print your name at the top of your card.

As you work the exam, lightly shade in the correct answers on your answer card. At the end of your exam when you are certain of all your choices darken all your answer boxes. If your card becomes damaged please ask your proctor for a new one.

Instructions for the hand graded part

Problems 14 and 15 are hand graded and each is worth 9 points. Write your answers on the test page. Show your work neatly and cross out irrelevant scratch work, false starts etc. A mere final answer will only get partial credit. You must show the complete work showing all steps.

Please put your name on each of the pages containing Problems 14 and 15, since they may be separated during grading. Also please add your Discussion Section Letter, which can be found on the first page of this exam, on both these pages.
1. Find the average value of the function \( f(x) = e^x \) in the interval \([0, \ln 2]\).

(A) \( \frac{2}{\ln 2} \)
(B) \( \frac{2}{\ln 2} - 1 \)
(C) \( \frac{2}{3\ln 2} \)
(D) \( \frac{1}{\ln 2} \)
(E) \( \frac{2}{\ln 2} + 1 \)
(F) \( \frac{1}{\ln 2} + 2 \)
(G) \( \frac{1}{\ln 2} + 1 \)
(H) \( \frac{2}{3\ln 2} + 2 \)
(I) \( \frac{2}{\ln 2} - 2 \)
(J) \( \frac{3}{3\ln 2} - 2 \)
2. A force of 10 N is required to hold a spring stretched 5 cm beyond its natural length. How much work is done in stretching from its natural length to 10 cm beyond its natural length?

(A) 1 Joule
(B) 2 Joules
(C) 3 Joules
(D) 4 Joules
(E) 5 Joules
(F) 6 Joules
(G) 7 Joules
(H) 8 Joules
(I) 9 Joules
(J) 10 Joules
3. A circular swimming pool has diameter 10 m and it is 2 m deep. If it is full of water, how much work is done in pumping all the water out from the top? (Gravitational force is 9.8 m/sec² and the density of water is 1000 kg/m³.)

(A) 49π Joules
(B) 49π × 10 Joules
(C) 49π × 10² Joules
(D) 49π × 10³ Joules
(E) 49π × 10⁴ Joules
(F) 49π × 10⁵ Joules
(G) 49π × 10⁶ Joules
(H) 49π × 10⁷ Joules
(I) 49π × 10⁸ Joules
(J) 49π × 10⁹ Joules
4. If \( f(x) = k \sin 2x \) for \( 0 \leq x \leq \pi/4 \) where \( k \) is a constant and zero if \( x < 0 \) or \( x > \pi/4 \) and it is a probability density function, what is \( k \)? For this value of \( k \) find its mean \( \mu \).

(A) \( k = 1, \mu = 1 \)
(B) \( k = 2, \mu = 1 \)
(C) \( k = 3, \mu = 1 \)
(D) \( k = 2, \mu = 2 \)
(E) \( k = 2, \mu = \frac{1}{2} \)
(F) \( k = 3, \mu = \frac{1}{2} \)
(G) \( k = 1, \mu = \frac{1}{2} \)
(H) \( k = 1, \mu = \frac{1}{3} \)
(I) \( k = 2, \mu = \frac{1}{3} \)
(J) \( k = 2, \mu = \frac{1}{4} \)
5. If \( f(x) \) is an exponential density function with mean \( \mu \) its median is,

(A) \( \mu \)
(B) \( \mu^2 \)
(C) \( \mu \ln 2 \)
(D) \( \mu^2 \ln 2 \)
(E) \( \frac{\mu}{2} \)
(F) \( \frac{\mu}{\ln 2} \)
(G) \( \frac{\mu^2}{2} \)
(H) \( \frac{\mu^2}{\ln 2} \)
(I) \( 2\mu \)
(J) \( 2\mu \ln 2 \)
6. Does the sequence \( a_n = \cos \frac{1}{n} + n \sin \frac{1}{n} \) converge? If it does what is its limit?

(A) The sequence diverges.
(B) Converges and the limit is zero.
(C) Converges and the limit is 1.
(D) Converges and the limit is 2.
(E) Converges and the limit is 3.
(F) Converges and the limit is \( \pi \).
(G) Converges and the limit is \( \pi/2 \).
(H) Converges and the limit is \( \frac{1}{2} \).
(I) Converges and the limit is \( \frac{\pi}{4} \).
(J) Converges and the limit can not be calculated.
7. Let a sequence be defined by the recursive formula, \( a_1 = 1, \ a_{n+1} = \frac{4 + 3a_n}{3 + 2a_n} \) for \( n \geq 1 \). If the sequence is convergent, its limit is,

(A) \( \frac{4}{3} \)

(B) \( \frac{3}{2} \)

(C) \( \frac{3}{4} \)

(D) \( 2 \)

(E) \( 1 \)

(F) \( \sqrt{2} \)

(G) \( \sqrt{3} \)

(H) \( \sqrt{\frac{1}{3}} \)

(I) \( \sqrt{\frac{3}{2}} \)

(J) \( \frac{1}{2} \)
8. Compute

\[ \sum_{n=1}^{\infty} \frac{1}{n(n+1)}. \]

(A) 0.1  
(B) 0.2  
(C) 0.4  
(D) 0.7  
(E) 1  
(F) 1.2  
(G) 1.3  
(H) 1.4  
(I) 1.9  
(J) 2
9. Compute

\[ \sum_{n=1}^{\infty} \frac{1+2^n}{3^n}. \]

(A) 1  
(B) 1.5  
(C) 2  
(D) 2.5  
(E) 3  
(F) 3.5  
(G) 4  
(H) 4.5  
(I) 5  
(J) 5.5
10. If $\sum_{n=2}^{\infty} (1 + c)^{-n} = 2$ then $c$ is,

(A) $\frac{1}{2}$

(B) $\sqrt{2}$

(C) $\sqrt{3}$

(D) $\sqrt{2} - 1$

(E) $\sqrt{3} - 1$

(F) $\frac{\sqrt{2} - 1}{2}$

(G) $\frac{\sqrt{3} - 1}{2}$

(H) $\frac{\sqrt{2} - 1}{3}$

(I) $\frac{\sqrt{3} - 1}{3}$

(J) $\frac{\sqrt{2} - 1}{5}$
These are three TRUE or FALSE questions each worth 4 points.

11. The series
\[ \sum_{n=1}^{\infty} \sin \left( \frac{1}{n^2} \right) \]
converges.

(A) TRUE
(B) FALSE
12. The series

\[ \sum_{n=1}^{\infty} (-1)^{n-1} \frac{n + 1}{n^2} \]

converges.

(A) TRUE
(B) FALSE
13. Let $X$ be the continuous random variable corresponding to the money a person makes in a year. Mr. D. Trump says that the chances of someone making more than a million dollars is one in four. He also says that the chances of someone making less than a hundred thousand dollars is five in six. Can he be telling the truth?

(A) YES
(B) NO