1. Using Newton's method to find the square root of \( x = 354.5 \) by solving the equation \( x^2 - 354.5 = 0 \) beginning at \( x = 82 \), give the value of \( x \) that you obtain in the first step.

\[
y = 82^2 - 354.5 = 6369.5
\]
\[
dy/dx = 2x = 164
\]
\[
\text{new } x = 82 - 6369.5/164 = 43.1646
\]

2. Find \( \int_{13}^{29} \sqrt{29x + 40} \) \( x \) \( dx \).

\[
\begin{align*}
  u & = 29x + 40, \quad x = (u - 40)/29 \\
  du & = 29dx, \quad dx = 1/29 du \\
  \text{when } x & = 29, \ u = 29 \times 29 + 40 = 881 \\
  \text{when } x & = 13, \ u = 29 \times 13 + 40 = 417
\end{align*}
\]
\[
\begin{align*}
  & = \frac{1}{29} \int_{417}^{881} u^{1/2} (u - 40) \ du \\
  & = \frac{1}{841} \left[ \frac{2}{5} u^{5/2} - 40 \times \frac{2}{3} u^{3/2} \right]_{417}^{881} \\
  & = \frac{1}{841} \left( 9215094 - 697321 - 1420366 + 227077 \right) = 7324484/841 = 8709
\end{align*}
\]
3. Find \( \int_{5.43}^{8.76} \sqrt{\ln x + 3.9} / x \, dx \).

\[
\begin{align*}
u &= \ln x, \quad du = 1/x \, dx \\
\text{when } x = 8.76, \quad u &= 2.1702 \\
\text{when } x = 5.43, \quad u &= 1.6919 \\
\int_{1.6919}^{2.1702} (u + 3.9)^{1/2} \, du &= \frac{2}{3} \left[ (u^{3/2}) \right]_{1.6919}^{2.1702} \\
&= \frac{2}{3} \left( 14.9556 - 13.2234 \right) = 1.1548
\end{align*}
\]

4. Under the continuous model, with \$6,900\ savings per year, spread uniformly over each year, how much is in a person’s retirement account after 40 years if the interest rate is 6.32 percent per year compounded continuously?

\[
(6900/0.0632) \left( e^{0.0632 \times 40} - 1 \right) = 109177.22 \times 11.5284 = \$1,258,641.25
\]
5. Suppose you are at a nice beach in Florida. The waves are 1.81 feet high and 52.58 feet apart. The water depth is 4.59 feet. How fast in feet per second are the waves traveling?

\[
\sqrt{\frac{32.2 \times 52.58}{2 \pi}} \cdot \tanh \left( \frac{2 \pi \times 4.59}{52.58} \right) = \sqrt{269.46 \times 0.4994} = 11.600
\]

6. A particle moves around a circle of radius 7.92 ft at the constant speed of 8.97 ft/sec. Determine the magnitude of its acceleration at any position.

Since the speed is constant, \( |\mathbf{v}| = 0 \), so \( |\mathbf{a}| = |\mathbf{a}_n| = |\mathbf{v}|^2 / R = 8.97^2 / 7.92 = 10.159 \)
7. If you have a 15-year amortized home mortgage for the amount of $149,500 at an annual interest rate of 5.2%, how much will each monthly payment be?

Since the payments are made monthly, there are $15 \times 12 = 180$ payments, and the interest rate for each month is $0.052/12 = 0.00433$. By our “false position” method, we first calculate the present value of 180 monthly payments of one dollar each:

$$\left(1 - \frac{1}{1.00433}\right)^{180} / 0.00433 = 0.54082 / 0.00433 = 124.8048$$

We then divide the loan amount by this present value to determine what the 180 monthly payments must be to equal the present value of the loan:

$$\frac{149,500}{124.8048} = 1197.87$$

8. Express $e^{-1.6x}$ as a power series about 0. Sum the first three terms of this series for the value $x = 3.69$.

The first three terms of the series are: $1 + \frac{(-1.6x)^1}{1!} + \frac{(-1.6x)^2}{2!}$. Substituting $x = 3.69$, we have:

$$1 + \frac{-5.904}{1!} + \frac{34.857}{2!} = 1 - 5.904 + 17.429 = 12.525.$$
9. Find the first three nonzero terms of the Maclaurin series for $e^{2x} \cos 5x$. Evaluate their sum for the value $x = 2.7$.

$$
(1 + \frac{(3x)^3}{3!} + \frac{(3x)^5}{5!} + \cdots) \times (1 - \frac{(5x)^3}{3!} + \cdots) = 1 + 3x + (9x^2/2 - 25x^2/2) + \text{higher degree terms.}
$$

This simplifies to $1 + 3x - 8x^2$, which equals $1 + 8.1 - 58.32 = -49.22$.

10. A surveyor satellite is in a circular orbit around a distant planet very much like Earth. If the period of its orbit is 17 of our Earth hours, and the radius of its orbit is 15000 miles, how many feet does it fall back to the planet each second in maintaining its circular orbit? (Use any of the methods discussed in the Orbits and How Could Isaac Have Done It lectures.)

From the Orbits lecture:
The satellite is $15000 \times 5280 = 79,200,000$ feet from the planet's center. In one second, the satellite moves through $\theta = 2\pi/(17 \times 60 \times 60) = 0.000102664266$ radians of arc. If it were not for the planet's gravity pulling the satellite into the circular orbit, in one second it would be $79,200,000/\cos \theta = 79,200,000.4174$ feet away from the planet's center (flying away along a straight line). Therefore, it falls back toward the planet 0.4174 feet each second in order to maintain the circular orbit. (This method has a huge loss of accuracy, but it is tolerable since the satellite is a lot closer to the planet than the Moon is to our Earth.)

From the How Could Isaac Have Done It lecture:
He could have used the Maclaurin series for $\sec \theta$ or the binomial expansion of $\sqrt{1 + \theta^2}$. In either case, the first two terms produce a good approximation to the fall-back as:

$$
R \times \frac{\theta^2}{2} = 79,200,000 \times 0.000102666^2/2 = 0.4174
$$

This is the same value we obtained dividing by $\cos \theta$ as in the Orbits lecture.