Math 233 Exam I Spring 2001

This examination has 30 multiple choice questions. Please check it over and if you find it to be incomplete, notify the proctor. Do all your supporting calculations in this booklet. In case of a doubful mark on your answer card, we can then check here. When you mark your card, use a soft lead pencil (#2). Erase fully any answers you want to change. Each problem is worth 1 point.

You may use a scientific calculator, such as the TI-83, but not any calculator that has a CAS. You may also use both sides of a 3×5 notecard which you have prepared beforehand with notes and formulas.

1. What is the set of points in R^3 satisfying the equation $x^2 + z^2 = 4$?

A) A circle of radius 2 in the xz-plane.

B) A circle of radius 4 in the xz-plane.

C) A cylinder of radius 2 around the y-axis.

D) A cylinder of radius 4 around the y-axis.

E) A sphere of radius 2 about the origin.

F) A sphere of radius 4 about the origin.

G) The surface of a cube about the origin with sidelength 2.

H) The surface of a cube about the origin with sidelength 4.

I) The set of points $(\pm 2, 0, 0)$, $(0, \pm 2, 0)$, $(0, \pm 2, 0)$.

J) the entire xz coordinate plane.

2. Find the center and radius, r, of the sphere $x^2 + y^2 + z^2 - 2x - 4z = 20$.

A) $(2, 0, 4),\ r = \sqrt{5}$

B) $(1, 2, 0),\ r = \sqrt{5}$

C) $(1, 0, 2),\ r = 5$

D) $(-1, 0, -2),\ r = 5$

E) $(2, 0, 4),\ r = \sqrt{20}$

F) $(-2, 0, -4),\ r = \sqrt{20}$

G $(1, 0, 2),\ r = 20$

H) $(-2, 0, -4),\ r = 20$

I) $(-1, 0, -2),\ r = 25$

J) $(2, 0, 4),\ r = 25$

3. Let $\boldsymbol{a} = \langle 2, 3, 1 \rangle$ and $\boldsymbol{b} = \langle -2, 5, -3 \rangle$. Find $|\boldsymbol{a} + \boldsymbol{b}|$.

A) 5

B) $\sqrt{28}$

C) 6

D) $\sqrt{40}$

E) 7

F) $\sqrt{52}$

G) 8

H) $\sqrt{68}$

I) 52

J) 68

4. Let $\boldsymbol{a} = \boldsymbol{i} + \boldsymbol{j}$ and $\boldsymbol{b} = \boldsymbol{j} - \boldsymbol{k}$. Describe the set of points (x, y, z) in R^3 where

$$\langle x, y, z \rangle = \boldsymbol{k} + s\boldsymbol{a} + t\boldsymbol{b}$$

where s and t are real numbers.

A) A point

B) A line through $(1, 1, 0)$ in the direction $\boldsymbol{j} - \boldsymbol{k}$

C) A line through the points $(1, 1, 0)$ and $(0, 1, -1)$

D) A line through the origin in the direction $-\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$

E) Two parallel lines

F) Two nonparallel lines

G) A plane that goes through the origin

H) A plane that does not go through the origin

I) The empty set (no such (x, y, z) exist)

J) All of R^3

5. Let $\boldsymbol{a} = \boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}$ and let $\boldsymbol{b} = 6\boldsymbol{i} - 2\boldsymbol{j} - 3\boldsymbol{k}$. Find $\boldsymbol{a} \cdot (\boldsymbol{a} - \boldsymbol{b})$.

A) -12

B) -1

C) 0

D) 1

E) 8

F) $5\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$

G) $-5\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$

H) $-6\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$

I) $6\boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}$

J) $84\boldsymbol{i} - 28\boldsymbol{j} - 42\boldsymbol{k}$

6. Find the angle between the vectors $\boldsymbol{a} = \langle 3, 1, 4 \rangle$ and $\boldsymbol{b} = \langle 4, -3, 1 \rangle$.

A) 0

B) $\pi/6$

C) $\pi/3$

D) $\pi/2$

E) $2\pi/3$

F) $5\pi/6$

G) π

H) $3\pi/2$

I) $cos^{-1}(13)$

7. Let $\boldsymbol{a} = -3\boldsymbol{i} + 5\boldsymbol{j} - 7\boldsymbol{k}$ and let $\boldsymbol{b} = \boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$. Find the scalar projection, $\text{comp}_b \boldsymbol{a}$, of $\boldsymbol{a}$ onto $\boldsymbol{b}$.

A) -14

B) $-\sqrt{14}$

C) -3

D) $-\sqrt{7}$

E) 0

F) $\sqrt{7}$

G) 3

H) $\sqrt{14}$

I) 14

8. Let $\boldsymbol{a} = \langle 3, 2, -4 \rangle$ and let $\boldsymbol{b} = \langle 2, -5, 1 \rangle$. Find $\boldsymbol{a} \times \boldsymbol{b}$.

A) $\langle -18, -5, 19 \rangle$

B) $\langle -18, -11, -19 \rangle$

C) $\langle 18, -5, 11 \rangle$

D) $\langle 18, -11, 11 \rangle$

E) $\langle 18, 11, -19 \rangle$

F) $\langle -22, -5, 19 \rangle$

G) $\langle -22, 5, -19 \rangle$

H) $\langle -22, 5, 11 \rangle$

I) $\langle 22, -11, 11 \rangle$

J) $\langle 22, 11, 19 \rangle$

9. Find a <u>unit</u> vector perpendicular to both the vector $\boldsymbol{i} - \boldsymbol{j}$ and the vector $\boldsymbol{j} + 2\boldsymbol{k}$.

A) $\frac{2}{3}\boldsymbol{i} + \frac{2}{3}\boldsymbol{j} + \frac{1}{3}\boldsymbol{k}$

B) $\frac{2}{3}\boldsymbol{i} + \frac{2}{3}\boldsymbol{j} - \frac{1}{3}\boldsymbol{k}$

C) $\frac{2}{3}\boldsymbol{i} - \frac{2}{3}\boldsymbol{j} + \frac{1}{3}\boldsymbol{k}$

D) $\frac{2}{3}\boldsymbol{i} - \frac{2}{3}\boldsymbol{j} - \frac{1}{3}\boldsymbol{k}$

E) $\frac{2}{5}\boldsymbol{i} + \frac{2}{5}\boldsymbol{j} + \frac{1}{5}\boldsymbol{k}$

F) $\frac{2}{5}\boldsymbol{i} + \frac{2}{5}\boldsymbol{j} - \frac{1}{5}\boldsymbol{k}$

G) $\frac{2}{5}\boldsymbol{i} - \frac{2}{5}\boldsymbol{j} + \frac{1}{5}\boldsymbol{k}$

H) $\frac{2}{5}\boldsymbol{i} - \frac{2}{3}\boldsymbol{j} - \boldsymbol{k}$

I) $\frac{2}{5}\boldsymbol{i} - \frac{2}{3}\boldsymbol{j} + \boldsymbol{k}$

J) $2\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$

10. Find the volume of the parallelopiped spanned by $\langle 1,1,2\rangle$, $\langle 1,3,1\rangle$ and $\langle 4,1,1\rangle$.

A) -17

B) -6

C) -4

D) 2

E) 4

F) 6

G) 11

H) 17

I) 18

J) 24

11. A vector equation for the line through the point $(1,0,6)$ and perpendicular to the plane $x + 3y + z = 5$ is given by:

A) $x - 1 = \frac{y}{3} = z - 6$

B) $(x-1) + \frac{y}{3} + (z-6) = 0$

C) $x + 6z = 0$

D) $\langle x,y,z\rangle = \langle 1,0,6\rangle + \langle 1,3,1\rangle x$

E) $\langle x,y,z\rangle = \langle 1,3,1\rangle + \langle 1,0,6\rangle x$

F) There is no such line.

G) $\langle x,y,z\rangle = \langle 1,0,6\rangle + t\langle 1,3,1\rangle$

H) $\langle x,y,z\rangle \cdot \langle 1,0,6\rangle = 5$

I) $\langle x,y,z\rangle \cdot \langle 1,0,6\rangle = 0$

J) $\langle x,y,z\rangle \cdot \langle 1,3,1\rangle = 5$

12. Find the equation of the plane that passes through the origin and contains the points $(1,1,1)$ and $(1,1,0)$.

A) $z = -x - y$

B) $y = x$

C) $z = 0$

D) $(x-1) + (y-1) + (z-1) = (x-1) + (y-1)$

E) $\langle x, y, z \rangle \cdot \langle 1,1,1 \rangle = \langle 1,1,0 \rangle$

F) $z = y$

G) $z = x$

H) $x + y = 0$

I) 17

J) $ax + by + cz = 0$

13. Find the distance between the parallel planes given by the equations:

$$z = x + 2y + 1, \text{ and}$$
$$3x + 6y - 3z = 4.$$

A) 12

B) 16

C) 0

D) $\frac{1}{\sqrt{2}} \left(= \frac{\sqrt{2}}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \right)$

E) $\sqrt{3}/2$

F) $7\sqrt{6}$

G) $7\sqrt{6}/18$

H) 1

I) $7\sqrt{3}/18$

J) $\sqrt{6}$

14. Find a normal vector to the plane that contains the x-axis and is parallel to the vector $\langle 2,2,1\rangle$.

A) $\langle 0,0,1\rangle$

B) $(0,0,0)$

C) $(2,2,1)$

D) $(1,2,2)$

E) $(1,0,0)$

F) $\langle 1,0,0\rangle$

G) $\langle 0,1,1\rangle$

H) π

I) $\langle 0,-1,2\rangle$

J) $\langle 1,1,2\rangle$

15. Find the domain of the function $f(x,y)=\frac{\sqrt{x-y}}{x+y}$.

A) The whole plane.

B) $\{(x,y)|x\neq y\}$

C) $\{(x,y)|x\neq -y\}$

D) $\{(x,y)|x>y\}$

E) $\{(x,y)|x<y\}$

F) $\{(x,y)|x\neq -y, x\geq y\}$

G) $\{(x,y)|\, x\neq y, x\neq -y\}$

H) $\{(x,y)|x\neq -y, x<y\}$

I) $\{(x,y)|\, x>y, x\neq y\}$

J) $\{(x,y)|\, x=-y\}$

16. The domain of $f(x,y) = \begin{cases} \frac{1}{x-y} & \text{if } y \neq x \\ 15 & \text{if } y = x \end{cases}$

is:

A) $\{(x,y) | x \neq y\}$

B) $(-\infty, \infty)$

C) $(0, \infty)$

D) $(-\infty, 0)$

E) The whole plane $(\mathbb{R}^2)$

F) $\{(x,y) |\ x, y \neq 15\}$

G) $\{(x,y) | x \neq 15\}$

H) This is <u>not</u> a function.

I) None of the above.

17. Which of the following are traces (cross-sections) of the graph of $f(x, y) = y^2 - x^2$?

I $z = y^2 - 7$

II $z = y^2 - 4$

III $z = -x^2 + 4$

IV $y^2 - x^2 = 16$

A) I

B) II

C) III

D) IV

E) I, II, III, IV

F) I, II, III

G) II, III, IV

H) I, III, IV

I) I, II

J) III, IV

18. If (x, y) has a polar representation (r_0, θ_0) and $F(r_0, \theta_0) \neq 0$, which of the following could be true?

I (x, y) is on the graph of $F(r, \theta) = 0$

II (x, y) is not on the graph of $F(r, \theta) = 0$

A) I

B) II

C) Both I and II could be true.

D) Neither could be true.

19. Find a cartesian equation for the curve described by the polar equation

$$r = 1/(1 - \sin\theta).$$

A) $x^2 + y^2 = 1$

B) $y^2 = 1 + 2x$

C) $x^2 - y^2 = 1$

D) $y^2 = 2 + x$

E) $x^2 + 2y^2 = 1$

F) $x^2 = 1 + 2y$

G) $y^2 = 2 - x$

H) $x^2 - 2y^2 = 1$

I) $x + 2y^2 = 1$

J) $x^2 - 3y^2 = 1$

20. Find the exact length of the curve given in polar coordinates by $r = e^{-\theta}, 0 \leq \theta \leq \pi$.

A) 2π

B) $e^{-\pi}$

C) $e^{-\pi} - 2$

D) $\sqrt{2\pi}$

E) $\sqrt{3}(\pi - e^2)$

F) $e^{-\pi}/2$

G) $\sqrt{2}(1 - e^{-\pi})$

H) $\pi + e^{-\pi}$

I) $e^{\pi} + 3$

J) 3π

21. Point P has spherical coordinates $(\rho, \theta, \phi) = (3, 0, \pi)$. Point Q has cylndrical coordinates $(r, \theta, z) = (3, 0, -6)$. What is the square of the distance between P and Q ?

A) 13

B) 14

C) 15

D) 16

E) 17

F) 18

G) 19

H) 20

I) 21

J) 22

22. The parabola $z = 4y^2$ is rotated about the z-axis. Let $z = f(r)$ be the equation of the resulting surface of revolution in cylindrical coordinates. What is $f(2)$?

A) 14

B) 15

C) 16

D) 17

E) 18

F) 1

G) 2

H) 3

I) 4

J) 5

23. $\boldsymbol{r}(t) = \langle 1, t, t^2 \rangle$. What is the square of the z-component of $\boldsymbol{T(1)}$?

A) 0.8

B) 0.7

C) 0.6

D) 0.5

E) -0.1

F) -0.2

G) -0.3

H) -0.4

I) -0.5

J) 0

24. If $\boldsymbol{r}'(t) = \langle \sin t, -\cos t, 2t \rangle$ with $\boldsymbol{r}(0) = \langle 1, 1, 2 \rangle$. What is the y-component of $\boldsymbol{r}(\pi/2)$?

A) $\sqrt{2}$

B) $\sqrt{2}/2$

C) $-\sqrt{2}$

D) $-\sqrt{2}/2$

E) 0

F) $\sqrt{3}/2$

G) $-\frac{1}{2}$

H) $\frac{1}{2}$

I) $-\sqrt{3}/2$

J) None of the above.

25. $\boldsymbol{r}(t) = \langle t, t^2, t^3 \rangle$. Let κ be the curvature of the curve. What is $(\kappa)^2$ at $t = 1$?

A) $\frac{34}{29}$

B) $\frac{49}{62}$

C) $\frac{76}{(14)^3}$

D) $\frac{63}{12}$

E) 1

F) 0

G) $\frac{25}{12}$

H) $\frac{12}{25}$

I) $\frac{29}{34}$

J) 6

26. $\boldsymbol{r}(t) = \langle e^t, e^t sin\, t, e^t cos\, t\rangle$. If the equation of the normal plane at $(1,0,1)$ is $ax + by + cz = 1$, what is $a + b + c$?

A) 0

B) -1

C) -2

D) -4

E) -7

F) 1.5

G) 2.3

H) 3.2

I) 4

J) 7

27. $\boldsymbol{r}(t) = \langle 1 + 2t, 1 + t - t^2, 1 - t + t^2 - t^3\rangle$. The tangent line to the curve at $t = 0$ intersects the x-y plane at a point with x-coordinate A and y-coordinate B. What is $A + 2B$?

A) 12

B) 11

C) 10

D) 9

E) 8

F) 7

G) 6

H) 5

I) 4

J) 3

28. $\boldsymbol{r}(t) = \langle t^2, 2t, ln\, t\rangle$, $1 \leq t \leq e$. If the length of the curve is e^a, what is a ?

A) -4

B) -3

C) -2

D) -1

E) 4

F) 3

G) 2

H) 1

I) 0

J) None of the above.

29. Find the curvature of the curve of intersection of the cylinder $x^2 + y^2 = 16$ and the plane $x + z = 5$ at $(4, 0, 1)$.

A) $\sqrt{5} + \sqrt{2}$

B) $\sqrt{5}/3$

C) $\sqrt{2}/4$

D) 0

E) $1/2$

F) $-1/3$

G) $(3)^{\frac{1}{3}}$

H) $4^{\frac{1}{3}}$

I) $5^{\frac{1}{3}}$

J) $\sqrt{5}$

30. Let $k(x)$ be the curvature of $y = ln\, x$ at x. Find $\lim_{x \to \infty} k(x)$.

A) e

B) e^{-1}

C) $ln2$

D) 1

E) 2

F) 0

G) -1

H) -2

I) $-e$

J) ∞